PROJECTILE MOTION

1. BASIC CONCEPT :

1.1 Projectile

Any object that is given an initial velocity obliquely, and that subsequently follows a path determined by the net constant force, (In this chapter constant force is gravitational force) acting on it is called a projectile.

Examples of projectile motion :

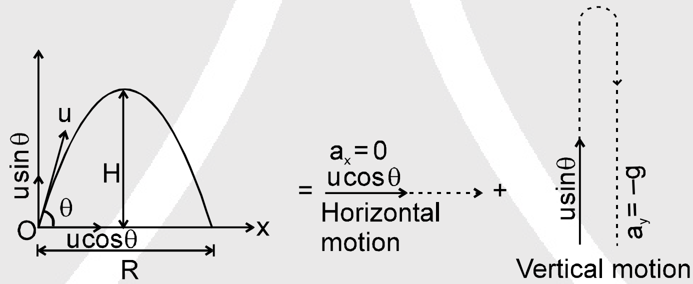
- A cricket ball hit by the batsman for a six
- A bullet fired from a gun.
- A packet dropped from a plane; but the motion of the aeroplane itself is not projectile motion because there are forces other than gravity acting on it due to the thrust of its engine.

1.2 Assumptions of Projectile Motion :

- We shall consider only trajectories that are of sufficiently short range so that the gravitational force can be considered constant in both magnitude and direction.
- All effects of air resistance will be ignored.
- Earth is assumed to be flat.

1.3 Projectile Motion :

- The motion of projectile is known as projectile motion.
- It is an example of two dimensional motion with constant acceleration.
- Projectile motion is considered as combination of two simultaneous motions in mutually perpendicular directions which are completely independent from each other i.e. horizontal motion and vertical motion.

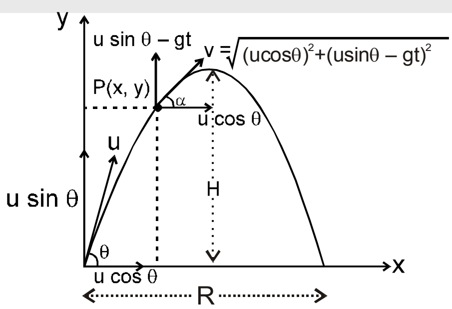


Parabolic path = vertical motion + horizontal motion.

Galileo's Statement :

Two perpendicular directions of motion are independent from each other. In other words any vector quantity directed along a direction remains unaffected by a vector perpendicular to it.

2. PROJECTILE THROWN AT AN ANGLE WITH HORIZONTAL



- Consider a projectile thrown with a velocity u making an angle θ with the horizontal.
- Initial velocity u is resolved in components in a coordinate system in which horizontal direction is taken as x-axis, vertical direction as y-axis and point of projection as origin.

$u_x = u\cos\theta$ $\qquad$ $u_y = u\sin\theta$

- Again this projectile motion can be considered as the combination of horizontal and vertical motion. Therefore,

Horizontal direction
(a) Initial velocity $u_x = u \cos \theta$
(b) Acceleration $a_x = 0$
(c) Velocity after time t, $v_x = u \cos \theta$

Vertical direction
Initial velocity $u_y = u \sin \theta$
Acceleration $a_y = g$
Velocity after time t, $v_y = u \sin \theta - gt$

2.1 Time of flight :

The displacement along vertical direction is zero for the complete flight.
Hence, along vertical direction net displacement = 0

$\Rightarrow (u \sin \theta)\, T - \frac{1}{2} g T^2 = 0 \quad \Rightarrow \quad T = \frac{2u\sin\theta}{g}$

2.2 Horizontal range :

$R = u_x . T \quad \Rightarrow \quad R = u \cos \theta . \frac{2u\sin\theta}{g}$

$R = \frac{u^2 \sin 2\theta}{g}$

2.3 Maximum height :

At the highest point of its trajectory, particle moves horizontally, and hence vertical component of velocity is zero.
Using 3rd equation of motion i.e. $v^2 = u^2 + 2as$
we have for vertical direction

$0 = u^2 \sin^2 \theta - 2gH \quad \Rightarrow \quad H = \frac{u^2 \sin^2 \theta}{2g}$

2.4 Resultant velocity :

$\vec{v} = v_x \hat{i} + v_y \hat{j} = u \cos \theta\, \hat{i} + (u \sin \theta - gt)\, \hat{j}$

Where, $|\vec{v}| = \sqrt{u^2 \cos^2 \theta + (u \sin\theta - gt)^2}$ and $\tan \alpha = v_y / v_x$.

Also, $v\cos\alpha = u\cos\theta \quad \Rightarrow \quad v = \frac{u\cos\theta}{\cos\alpha}$

Note :

- Results of article 2.1, 2.2, and 2.3 are valid only if projectile lands at same horizontal level from which it was projected.
- Vertical component of velocity is positive when particle is moving up and vertical component of velocity is negative when particle is coming down if vertical upwards direction is taken as positive.

2.5 General result :

- For maximum range $\theta = 45°$
  $R_{max} = u^2/g \quad \Rightarrow \quad H_{max} = R_{max}/2$
- We get the same range for two angle of projections α and $(90 - \alpha)$ but in both cases, maximum heights attained by the particles are different.

  This is because, $R = \frac{u^2 \sin 2\theta}{g}$, and $\sin 2(90 - \alpha) = \sin 180 - 2\alpha = \sin 2\alpha$
- If R = H
  i.e. $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g} \quad \Rightarrow \quad \tan \theta = 4$
- Range can also be expressed as $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u\sin\theta . u\cos\theta}{g} = \frac{2u_x u_y}{g}$

Solved Example

Example 1. A body is projected with a speed of 30 ms^{-1} at an angle of 30º with the vertical. Find the maximum height, time of flight and the horizontal range of the motion. [Take g = 10 m/s^2]

Solution : Here u = 30 ms^{-1}, Angle of projection, θ = 90 – 30 = 60º

Maximum height, H = $\frac{u^2 \sin^2\theta}{2g} = \frac{30^2 \sin^2 60^o}{2\times10} = \frac{900}{20}\times\frac{3}{4} = \frac{135}{4}$ m

Time of flight, T = $\frac{2u\sin\theta}{g} = \frac{2\times30\times\sin60^0}{10} = 3\sqrt{3}$ sec.

Horizontal range = R = $\frac{u^2\sin2\theta}{g} = \frac{30\times30\times2\sin60^o\cos60^o}{10} = 45\sqrt{3}$ m

Example 2. A projectile is thrown with a speed of 100 m/s making an angle of 60° with the horizontal. Find the minimum time after which its inclination with the horizontal is 45° ?

Solution : u_x = 100 × cos60° = 50

u_y = 100 × sin60° = 50 $\sqrt{3}$

$v_y = u_y + a_y t = 50\sqrt{3} - gt$ and $v_x = u_x = 50$

When angle is 45°,

tan $45^0 = \frac{v_y}{v_x}$ $\Rightarrow$ $v_y = v_x$

$\Rightarrow$ $50 - gt\sqrt{3} = 50$ $\Rightarrow$ $50(\sqrt{3}-1) = gt$ $\Rightarrow$ $t = 5(\sqrt{3}-1)$ s

Example 3. A large number of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread ?

Solution : Maximum distance up to which a bullet can be fired is its maximum range, therefore

$R_{max} = \frac{v^2}{g}$

Maximum area = $\pi(R_{max})^2 = \frac{\pi v^4}{g^2}$.

Example 4. The velocity of projection of a projectile is given by : $\vec{u} = 5\hat{i} + 10\hat{j}$. Find

(a) Time of flight,
(b) Maximum height,
(c) Range

Solution : We have u_x = 5 u_y = 10

(a) Time of flight = $\frac{2u\sin\theta}{g} = \frac{2u_y}{g} = \frac{2\times10}{10}$ = 2 s

(b) Maximum height = $\frac{u^2\sin^2\theta}{2g} = \frac{{u_y}^2}{2g} = \frac{10\times10}{2\times10}$ = 5 m

(c) Range = $\frac{2u\sin\theta . u\cos\theta}{g} = \frac{2u_x u_y}{g} = \frac{2\times10\times5}{10}$ = 10 m

Example 5. A particle is projected at an angle of 30° w.r.t. horizontal with speed 20 m/s :

(i) Find the position vector of the particle after 1s.
(ii) Find the angle between velocity vector and position vector at t = 1s.

Solution :

(i) x = u cos θ t = 20 × $\frac{\sqrt{3}}{2}$ × t = 10 $\sqrt{3}$ m

y = u sin θt – $\frac{1}{2}$ × 10 × t^2 = 20 × $\frac{1}{2}$ × (1) – 5 $(1)^2$ = 5m

Position vector, $\vec{r} = 10\sqrt{3}\ \hat{i} + 5\hat{j}$, $|\vec{r}| = \sqrt{(10\sqrt{3})^2 + 5^2}$

(ii) $v_x = 10\sqrt{3}\ \hat{i}$

$v_y = u_y + a_y t = 10 - g\,t = 0$

$\therefore\quad \vec{v} = 10\sqrt{3}\ \hat{i}\ ,\ |\vec{v}| = 10\sqrt{3}$

$\vec{v}\cdot\vec{r} = (10\sqrt{3}\hat{i})\cdot(10\sqrt{3}\hat{i}+5\hat{j}) = 300$

$\vec{v}\,.\,\vec{r} = |\vec{v}|\,|\vec{r}|\cos\theta$

$$\Rightarrow \cos\theta = \frac{\vec{v}\cdot\vec{r}}{|\vec{v}||\vec{r}|} = \frac{300}{10\sqrt{3}\sqrt{325}} \quad\Rightarrow\quad \theta = \cos^{-1}\left(2\sqrt{\frac{3}{13}}\right)$$

3. EQUATION OF TRAJECTORY

The path followed by a particle (here projectile) during its motion is called its **Trajectory**. Equation of trajectory is the relation between instantaneous coordinates (Here x & y coordinate) of the particle.

If we consider the horizontal direction,

$x = u_x.t$

$x = u\cos\theta.\,t$ .....(1)

For vertical direction :

$y = u_y\,.\,t - 1/2\,gt^2$

$= u\sin\theta\,.\,t - 1/2\,gt^2$ .....(2)

Eliminating 't' from equation (1) & (2)

$$y = u\sin\theta\,.\,\frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2 \Rightarrow\quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

This is an equation of parabola called as trajectory equation of projectile motion.

Other forms of trajectory equation :

- $y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2}$
- $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

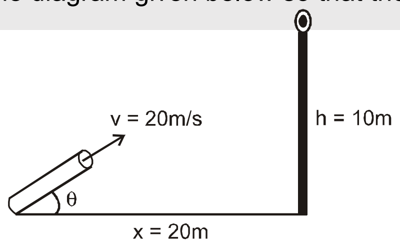
$$\Rightarrow\quad y = x\tan\theta\left[1-\frac{gx}{2u^2\cos^2\theta\tan\theta}\right]$$

$$\Rightarrow\quad y = x\tan\theta\left[1-\frac{gx}{2u^2\sin\theta\cos\theta}\right]$$

$$\Rightarrow\quad y = x\tan\theta\left[1-\frac{x}{R}\right]$$

Solved Example

Example 1. Find the value of θ in the diagram given below so that the projectile can hit the target.



Solution. $y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2} \quad\Rightarrow 10 = 20\tan\theta - \dfrac{5\times(20)^2}{(20)^2}(1+\tan^2\theta)$

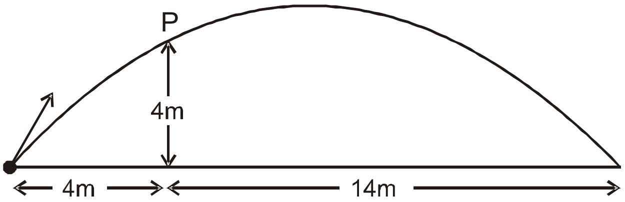
$\Rightarrow\quad 2 = 4\tan\theta - (1+\tan^2\theta)$

$\Rightarrow\quad \tan^2\theta - 4\tan\theta + 3 = 0$

$\Rightarrow\quad (\tan\theta - 3)(\tan\theta - 1) = 0 \quad\Rightarrow \tan\theta = 3, 1 \quad\Rightarrow\quad \theta = 45°, \tan^{-1}(3)$

Example 2. A ball is thrown from ground level so as to just clear a wall 4 m high at a distance of 4 m and falls at a distance of 14 m from the wall. Find the magnitude and direction of initial velocity of the ball figure is given below.

Solution.



The ball passes through the point P(4, 4). Also range = 4 + 14 = 18 m.

The trajectory of the ball is, $y = x \tan\theta\,(1 - \frac{x}{R})$

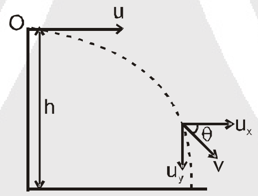
Now x = 4m, y = 4m and R = 18 m

$\therefore\ 4 = 4\tan\theta\left[1 - \frac{4}{18}\right] = 4\tan\theta.\ \frac{7}{9}$ or $\tan\theta = \frac{9}{7}$ $\Rightarrow$ $\theta = \tan^{-1}\frac{9}{7}$

And $R = \frac{2u^2\sin\theta\cos\theta}{g}$ or $18 = \frac{2}{9.8} \times u^2 \times \frac{9}{\sqrt{130}} \times \frac{7}{\sqrt{130}}$ $\Rightarrow$ $u = \sqrt{182}$

4. PROJECTILE THROWN PARALLEL TO THE HORIZONTAL FROM SOME HEIGHT

Consider a projectile thrown from point O at some height h from the ground with a velocity u. Now we shall study the characteristics of projectile motion by resolving the motion along horizontal and vertical directions.



Horizontal direction
(i) Initial velocity $u_x = u$
(ii) Acceleration $a_x = 0$

Vertical direction
Initial velocity $u_y = 0$
Acceleration $a_y = g$ (downward)

4.1 Time of flight :

This is equal to the time taken by the projectile to return to ground. From equation of motion

$S = ut + \frac{1}{2}at^2$, along vertical direction, we get

$-h = u_y t + \frac{1}{2}(-g)t^2$

$\Rightarrow\ h = \frac{1}{2}gt^2$

$\Rightarrow\ t = \sqrt{\frac{2h}{g}}$

4.2 Horizontal range :

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

$R = u_x\,.\,t \quad \Rightarrow \quad R = u\sqrt{\frac{2h}{g}}$

4.3 Velocity at a general point P(x, y) :

$v = \sqrt{u_x^2 + u_y^2}$

Here horizontal velocity of the projectile after time t

$v_x = u$

velocity of projectile in vertical direction after time t

$v_y = 0 + (-g)t = -gt = gt$ (downward)

$\therefore \quad v = \sqrt{u^2 + g^2t^2}$ and $\tan\theta = v_y/v_x$

4.4 Velocity with which the projectile hits the ground :

$V_x = u$

$V_y^2 = 0^2 - 2g(-h)$

$V_y = \sqrt{2gh}$

$V = \sqrt{V_x^2 + V_y^2} \quad \Rightarrow V = \sqrt{u^2 + 2gh}$

4.5 Trajectory equation :

The path traced by projectile is called the trajectory.

After time t,

$x = ut$ .....(1)

$y = \frac{-1}{2}gt^2$ .....(2)

From equation (1)

$t = x/u$

Put the value of t in equation (2)

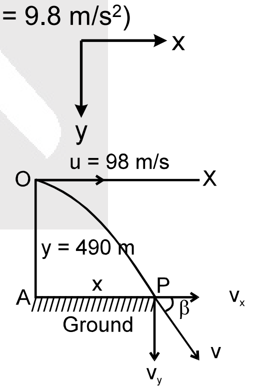
$y = \frac{-1}{2}g.\frac{x^2}{u^2}$

This is trajectory equation of the particle projected horizontally from some height.

Solved Example

Example 1. A projectile is fired horizontally with a speed of 98 ms^{-1} from the top of a hill 490 m high. Find
(i) the time taken to reach the ground
(ii) the distance of the target from the hill and
(iii) the velocity with which the projectile hits the ground. (take g = 9.8 m/s^2)

Solution :



(i) The projectile is fired from the top O of a hill with speed
u = 98 ms^{-1} along the horizontal as shown as OX.
It reaches the target P at vertical depth
OA, in the coordinate system as shown,
OA = y = 490 m
As, $y = 1/2\ gt^2$

$\therefore \quad 490 = \frac{1}{2} \times 9.8\ t^2$

or $t = \sqrt{100} = 10$ s.

(ii) Distance of the target from the hill is given by, AP = x = Horizontal velocity × time = 98 × 10 = 980 m.

(iii) The horizontal and vertical components of velocity v of the projectile at point P are

$v_x = u = 98\ ms^{-1}$

$v_y = u_y + gt = 0 + 9.8 \times 10 = 98\ ms^{-1}$

$V = \sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2}\ ms^{-1}$

Now if the resultant velocity v makes an angle β with the horizontal, then

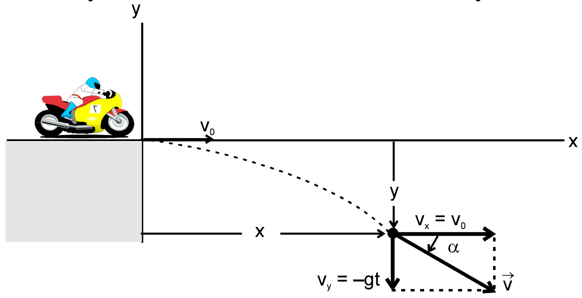
$\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \quad \therefore \quad \beta = 45^o$

Example 2. A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff and velocity after 0.5 s.

Solution : At t = 0.50 s, the x and y-coordinates are $x = v_0t = (9.0\ \text{m/s})\ (0.50\ \text{s}) = 4.5\ \text{m}$

$$y = -\frac{1}{2}gt^2 = -\frac{1}{2}\ (10\ \text{m/s}^2)\ (0.50\ \text{s})^2 = -\frac{5}{4}\text{m}$$

The negative value of y shows that this time the motorcycle is below its starting point.



The motorcycle's distance from the origin at this time $r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{5}{4}\right)^2} = \frac{\sqrt{349}}{4}$ m.

The components of velocity at this time are $v_x = v_0 = 9.0$ m/s

$v_y = -gt = (-10\ \text{m/s}^2)\ (0.50\ \text{s}) = -5$ m/s.

The speed (magnitude of the velocity) at this time is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.0\,\text{m/s})^2 + (-5\,\text{m/s})^2} = \sqrt{106}\ \text{m/s}$$

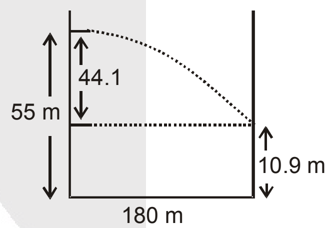
Example 3. An object is thrown between two tall buildings 180 m from each other. The object is thrown horizontally from a window 55 m above ground from one building through a window 10.9 m above ground in the other building. Find out the speed of projection. (Use g = 9.8 m/s^2)

Solution : $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 44.1}{9.8}}$

t = 3 sec.

R = uT

$\frac{180}{3} = u$; u = 60 m/s



5. PROJECTION FROM A TOWER

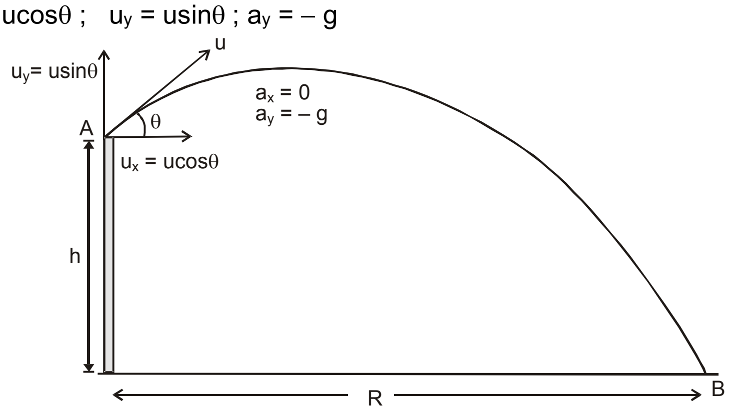
Case (i) : Horizontal projection

$u_x = u$; $u_y = 0$; $a_y = -g$

This is same as previous section (section 4)

Case (ii) : Projection at an angle θ above horizontal

$u_x = u\cos\theta$; $u_y = u\sin\theta$; $a_y = -g$



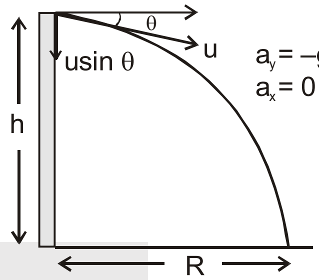
Equation of motion between A & B (in Y direction)
$S_y = -h$, $u_y = u\sin\theta$, $a_y = -g$, $t = T$

$S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -h = u\sin\theta\, t - \frac{1}{2} g t^2$

Solving this equation we will get time of flight, T.
And range, $R = u_x T = u\cos\theta\, T$; Also, $v_y^2 = u_y^2 + 2a_yS_y = u^2\sin^2\theta + 2gh$; $v_x = u\cos\theta$

$v_B = \sqrt{v_y^2 + v_x^2} \Rightarrow v_B = \sqrt{u^2 + 2gh}$

Case (iii) : Projection at an angle θ below horizontal



$u_x = u\cos\theta$; $u_y = -u\sin\theta$; $a_y = -g$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$S_y = -h$, $u_y = -u\sin\theta$, $t = T$, $a_y = -g$

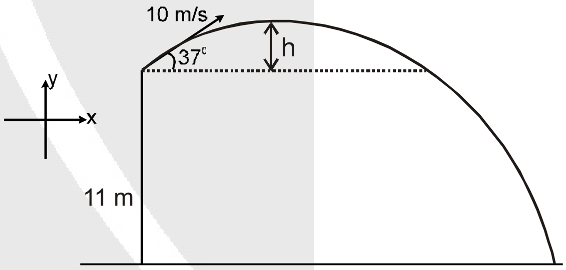
$\Rightarrow -h = -u\sin\theta\, T - \frac{1}{2} g T^2 \Rightarrow h = u\sin\theta\, T + \frac{1}{2} g T^2$

Solving this equation we will get time of flight, T.
And range, $R = u_x T = u\cos\theta\, T$
$v_x = u\cos\theta$
$v_y^2 = u_y^2 + 2a_yS_y = u^2\sin^2\theta + 2(-g)(-h)$
$v_y^2 = u^2\sin^2\theta + 2gh$
$v_B = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

Note : objects thrown from same height in different directions with same initial speed will strike the ground with the same final speed. But the time of flight will be different.

Solved Example

Example 1. From the top of a 11 m high tower a stone is projected with speed 10 m/s, at an angle of 37º as shown in figure. Find
(a) Speed after 2s
(b) Time of flight.
(c) Horizontal range.
(d) The maximum height attained by the particle.
(e) Speed just before striking the ground.



Solution :

(a) Initial velocity in horizontal direction = 10 cos 37 = 8 m/s
Initial velocity in vertical direction = 10 sin 37º = 6 m/s
Speed after 2 seconds
$v = v_x\hat{i} + v_y\hat{j} = 8\hat{i} + (u_y + a_yt)\hat{j} = 8\hat{i} + (6 - 10 \times 2)\hat{j} = 8\hat{i} - 14\hat{j}$

(b) $S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -11 = 6 \times t + \frac{1}{2} \times (-10)\, t^2$

$5t^2 - 6t - 11 = 0 \Rightarrow (t + 1)(5t - 11) = 0 \Rightarrow t = \frac{11}{5}$ sec.

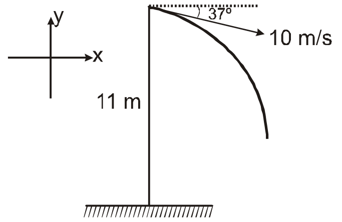
(c) Range $= 8 \times \frac{11}{5} = \frac{88}{5}$ m

(d) Maximum height above the level of projection, $h = \frac{u_y^2}{2g} = \frac{6^2}{2 \times 10} = 1.8$ m

$\therefore$

(e) Maximum height above ground = 11 + 1.8 = 12.8 m
$v = \sqrt{u^2 + 2gh} = \sqrt{100 + 2 \times 10 \times 11}$
$\Rightarrow v = 8\sqrt{5}$ m/s

Example 2. From the top of a 11 m high tower a stone is projected with speed 10 m/s, at an angle of 37º as shown in figure. Find
(a) Time of flight.
(b) Horizontal range.
(c) Speed just before striking the ground.



Solution : $u_x = 10 \cos 37^\circ = 8$ m/s , $u_y = -10 \sin 37^\circ = -6$ m/s

(a) $S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -11 = -6 \times t + \frac{1}{2} \times (-10) t^2$

$\Rightarrow 5t^2 + 6t - 11 = 0$

$\Rightarrow (t - 1)(5t + 11) = 0 \Rightarrow t = 1$ sec

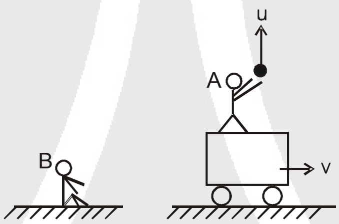
(b) Range = 8 × 1 = 8 m

(c) $v = \sqrt{u^2 + 2gh} = \sqrt{100 + 2 \times 10 \times 11}$

$\Rightarrow v = \sqrt{320}$ m/s $= 8\sqrt{5}$ m/s

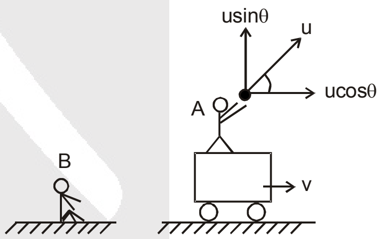
Note : that in Ex.11 and Ex.12, objects thrown from same height in different directions with same initial speed strike the ground with the same final speed, but after different time intervals.

6. PROJECTION FROM A MOVING PLATFORM

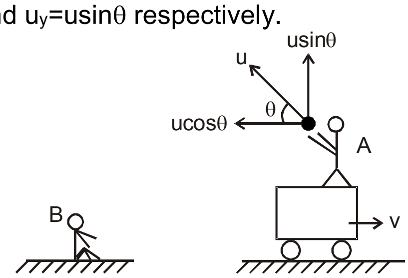


Case (i) : When a ball is thrown upward from a truck moving with uniform speed, then observer A standing in the truck, will see the ball moving in straight vertical line (upward & downward). The observer B sitting on road, will see the ball moving in a parabolic path. The horizontal speed of the ball is equal to the speed of the truck.

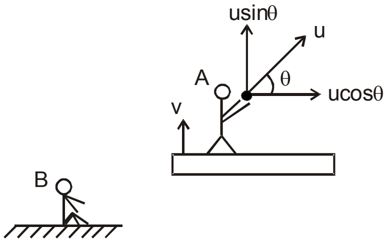
Case (ii) : When a ball is thrown at some angle 'θ' in the direction of motion of the truck, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck, is ucosθ, and usinθ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is
$u_x = u\cos\theta + v$ and $u_y = u\sin\theta$ respectively.



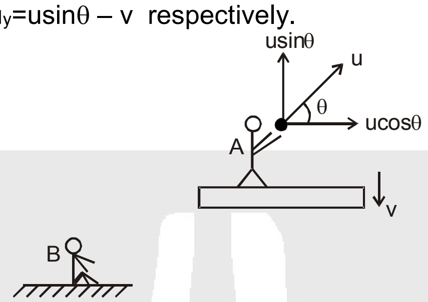
Case (iii) : When a ball is thrown at some angle 'θ' in the opposite direction of motion of the truck, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck, is ucosθ, and usinθ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is
$u_x = u\cos\theta - v$ and $u_y = u\sin\theta$ respectively.



Case (iv) : When a ball is thrown at some angle 'θ' from a platform moving with speed v upwards, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform, is ucosθ and usinθ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is u_x = ucosθ and u_y = usinθ + v respectively.



Case (v) : When a ball is thrown at some angle 'θ' from a platform moving with speed v downwards, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform, is ucosθ and usinθ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is u_x = ucosθ and u_y=usinθ – v respectively.



Solved Example

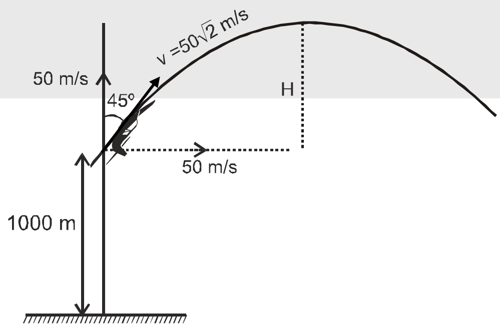
Example 1. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s² and the projection speed in the vertical direction is 9.8 m/s. How far behind the boy will the ball fall on the car ?

Solution : Let the initial velocity of car be 'u'.
time of flight, t = $2u_y/g$ = 2
where u_y = component of velocity in vertical direction
Distance travelled by car $x_c = u \times 2 + 1/2 \times 1 \times 2^2 = 2u + 2$
distance travelled by ball $x_b = u \times 2$
$x_c - x_b = 2u + 2 - 2u = 2$m **Ans.**

Example 2. A fighter plane moving with a speed of $50\sqrt{2}$ m/s upward at an angle of 45º with the vertical, releases a bomb. Find
(a) Time of flight
(b) Maximum height of the bomb above ground

Solution :

(a) $y = u_y t + \frac{1}{2} a_y t^2$



$-1000 = 50t - \frac{1}{2} \times 10 \times t^2$; $t^2 - 10t - 200 = 0$

$(t - 20)(t + 10) = 0$; t = 20 sec

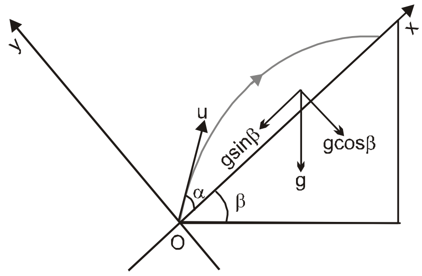
(b) $H = \frac{u_y^2}{2g} = \frac{50^2}{2g} = \frac{50 \times 50}{20} = 125$ m.

Hence maximum height above ground H = 1000 + 125 = 1125 m

7. PROJECTION ON AN INCLINED PLANE

Case (i) : Particle is projected up the incline

Here α is angle of projection w.r.t. the inclined plane. x and y axis are taken along and perpendicular to the incline as shown in the diagram.



In this case: $a_x = -\,g\sin\beta$

$u_x = u\cos\alpha$

$a_y = -\,g\cos\beta$

$u_y = u\sin\alpha$

7.1 Time of flight (T) :

When the particle strikes the inclined plane y becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2 \quad \Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Where $u_\perp$ and $g_\perp$ are component of u and g perpendicular to the incline.

7.2 Maximum height (H) :

When half of the time is elapsed y coordinate is equal to maximum distance from the inclined plane of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2 \quad \Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_\perp^2}{2g_\perp}$$

7.3 Range along the inclined plane (R):

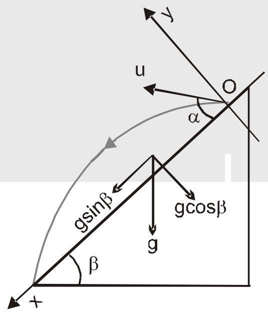
When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \quad R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2 \quad \Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$$

Case (ii) : Particle is projected down the incline

In this case :



$a_x = g\sin\beta$; $u_x = u\cos\alpha$

$a_y = -g\cos\beta$

$u_y = u\sin\alpha$

7.4 Time of flight (T) :

When the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2 \quad \Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

7.5 Maximum height (H) :

When half of the time is elapsed y coordinate is equal to maximum height of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2 \quad\Rightarrow\quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2_{\perp}}{2g_{\perp}}$$

7.6 Range along the inclined plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2}a_x t^2 \Rightarrow R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2 \Rightarrow R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$$

7.7 Standard results for projectile motion on an inclined plane

	Up the Incline	**Down the Incline**
Range	$\frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$	$\frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\frac{2u\sin\alpha}{g\cos\beta}$	$\frac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\frac{\pi}{4}-\frac{\beta}{2}$	$\frac{\pi}{4}+\frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

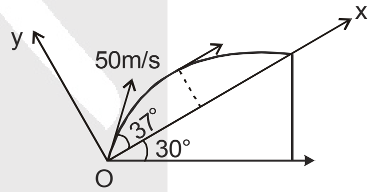
Here α is the angle of projection with the incline and β is the angle of incline.

Note : For a given speed, the direction which gives the maximum range of the projectile on an incline, bisects the angle between the incline and the vertical, for upward or downward projection.

Solved Example

Example 1. A bullet is fired from the bottom of the inclined plane at angle θ = 37° with the inclined plane. The angle of incline is 30° with the horizontal. Find
(i) The position of the maximum height of the bullet from the inclined plane.
(ii) Time of light
(iii) Horizontal range along the incline.
(iv) For what value of θ will range be maximum.
(v) Maximum range.

Solution :



(i) Taking axis system as shown in figure
At highest point $V_y = 0$
$V_y^2 = U_y^2 + 2a_y y$
$0 = (30)^2 - 2g\cos 30^\circ y$
$y = 30\sqrt{3}$ (maximum height) ......(1)

(ii) Again for x coordinate $V_y = U_y + a_y t$
$0 = 30 - g\cos 30^\circ \times t \quad\Rightarrow\quad t = 2\sqrt{3}$
$T = 2 \times 2\sqrt{3}$ sec Time of flight

(iii) $x = U_x t + \frac{1}{2}a_x t^2$

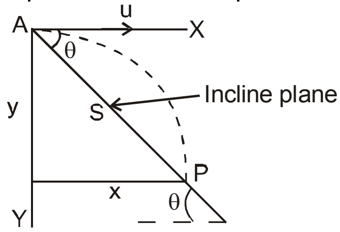
$x = 40\times 4\sqrt{3} - \frac{1}{2}g\sin 30^\circ \times (4\sqrt{3})^2 \quad\Rightarrow\quad x = 40\,(4\sqrt{3} - 3)$ m Range

(iv) $\frac{\pi}{4} - \frac{30^\circ}{2} = 45^\circ - 15^\circ = 30^\circ$

(v) $\frac{u^2}{g(1+\sin\beta)} = \frac{50\times 50}{10\left(1+\frac{1}{2}\right)} = \frac{2500}{15} = \frac{500}{3}$ m

Example 2. A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?

Solution : Take X, Y-axes as shown in figure. Suppose that the particle strikes the plane at a point P with coordinates (x, y). Consider the motion between A and P.



Suppose distance between A and P is S

Then position of P is,

$x = S\cos\theta$

$y = -S\sin\theta$

Using equation of trajectory (For ordinary projectile motion)

$y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

here $y = -S\sin\theta$

$x = S\cos\theta$

θ = angle of projection with horizontal = 0º

$-S\sin\theta = S\cos\theta\,(0) - \dfrac{g(S\cos\theta)^2}{2u^2}$ $\quad S = \dfrac{2u^2\sin\theta}{g\cos^2\theta}$

Aliter : $R = \dfrac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$ $\quad$ Here $\alpha = \beta = \theta$ $\quad \Rightarrow \quad R = \dfrac{2u^2\sin\theta}{g\cos^2\theta}$

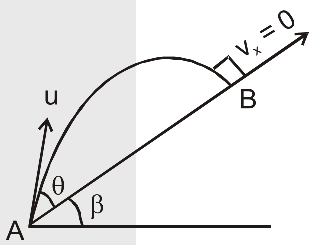
Example 3. A projectile is thrown at an angle θ with an inclined plane of inclination β as shown in figure. Find the relation between β and θ if :

(a) Projectile strikes the inclined plane perpendicularly, to the inclined plane

(b) Projectile strikes the inclined plane horizontally to the ground

Solution :

(a) If projectile strikes perpendicularly.



$v_x = 0$ when projectile strikes

$v_x = u_x + a_x t$

$0 = u\cos\theta - g\sin\beta T \quad \Rightarrow \quad T = \dfrac{u\cos\theta}{g\sin\beta}$

we also know that $T = \dfrac{2u\sin\theta}{g\cos\beta}$

$\Rightarrow \quad \dfrac{u\cos\theta}{g\sin\beta} = \dfrac{2u\sin\theta}{g\cos\beta} \quad \Rightarrow \quad 2\tan\theta = \cot\beta$

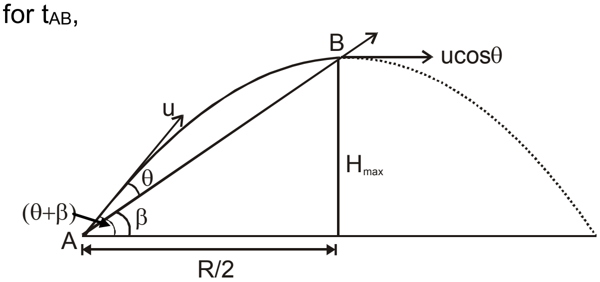
(b) If projectile strikes horizontally, then at the time of striking the projectile will be at the maximum height from the ground.

Therefore time taken to move from A to B, t_{AB} = 1/2 time of flight over horizontal plane

$= \dfrac{2u\sin(\theta+\beta)}{2\times g}$

Also, t_{AB} = time of flight over incline = $\dfrac{2u\sin\theta}{g\cos\beta}$

Equating for t_{AB},

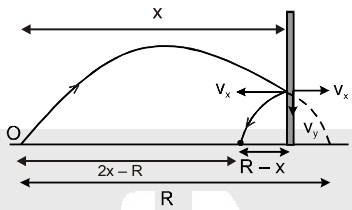


$\dfrac{2u\sin\theta}{g\cos\beta} = \dfrac{2u\sin(\theta+\beta)}{2g} \quad \Rightarrow \quad 2\sin\theta = \sin(\theta+\beta)\cos\beta.$

7.8 Elastic collision of a projectile with a wall :

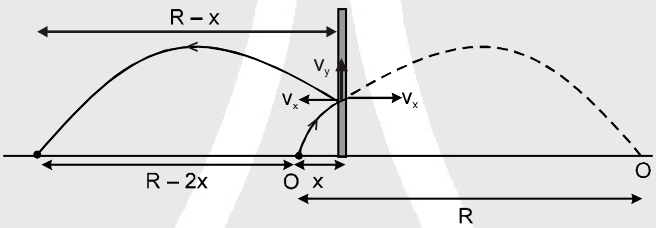
Suppose a projectile is projected with speed u at an angle θ from point O on the ground. Range of the projectile is R. A vertical, smooth wall is present in the path of the projectile at a distance x from the point O. The collision of the projectile with the wall is elastic. Due to collision, direction of x component of velocity is reversed but its magnitude remains the same and y component of velocity remains unchanged. Therefore the remaining distance (R – x) is covered in the backward direction and the projectile lands at a distance of R – x from the wall. Also time of flight and maximum height depends only on y component of velocity, hence they do not change despite of collision with the vertical, smooth and elastic wall.

Case (i) : If $x \geq \frac{R}{2}$



Here distance of landing place of projectile from its point of projection is 2x – R.

Case (ii) : If $x < \frac{R}{2}$



Here distance of landing place of projectile from its point of projection is R – 2x.

Solved Example

Example 1. A ball thrown from ground at an angle θ = 37º with speed u = 20 m/s collides with A vertical wall 18.4 meter away from the point of projection. If the ball rebounds elastically to finally fall at some distance in front of the wall, find for this entire motion,

(i) Maximum height

(ii) Time of flight

(iii) Distance from the wall where the ball will fall

(iv) Distance from point of projection, where the ball will fall.

Solution :

(i) $H = \frac{u^2 \sin^2 \theta}{2g} = \frac{(20)^2 \sin^2 37^\circ}{2 \times 10} = \frac{20 \times 20}{2 \times 10} \times \frac{3}{5} \times \frac{3}{5} = 7.2$ m

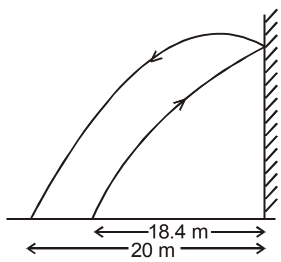
(ii) $T = \frac{2u \sin\theta}{g} = \frac{2 \times 20 \times \sin 37^\circ}{10} = 2.4$ sec.

(iii) $R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2}{g} \times 2 \sin\theta \cos\theta$

$\Rightarrow R = \frac{(20)^2}{10} \times 2 \sin 37^\circ \cos 37^\circ = 38.4$ m

Distance from the wall where the ball falls

= R – x = 38.4 – 18.4 = **20 m. Ans.**



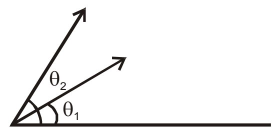
(iv) Distance from the point of projection = | R – 2x | = | 38.4 – 2 × 18.4|= 1.6 m

SOLVED MISCELLANEOUS PROBLEMS

Problem 1 Two projectiles are thrown with different speeds and at different angles so as to cover the same maximum height. Find out the sum of the times taken by each to the reach to highest point, if time of flight is T.

Answer : Total time taken by either of the projectile.

Solution : $H_1 = H_2$ (given)



$$\frac{u_1^2 \sin^2\theta_1}{2g} = \frac{u_2^2 \sin^2\theta_2}{2g}$$

$u_1^2 \sin^2\theta_1 = u_2^2 \sin^2\theta_2$ .....(1)

at maximum height final velocity = 0

$v^2 = u_1^2 - 2gH_1$

$U_1^2 = 2gH_1$ similarly $U_2^2 = 2gH_2$

$U_1 = U_2$

on putting in equation (1)

$\because\ u_1^2 \sin^2\theta_1 = u_2^2 \sin^2\theta_2 \Rightarrow \theta_1 = \theta_2$

$T_1 = \dfrac{2u_1 \sin\theta_1}{g} \Rightarrow T_2 = \dfrac{2u_2 \sin\theta_2}{g} \Rightarrow \therefore T_1 = T_2$

Time taken to reach the maximum height by 1st projectile = $\dfrac{T_1}{2}$

Time taken to reach the maximum height by 2nd projectile = $\dfrac{T_2}{2}$

$\therefore$ sum of time taken by each to reach highest point = $\dfrac{T_1}{2} + \dfrac{T_2}{2} = 2\dfrac{T_1}{2}$ (or $2\dfrac{T_2}{2}$) = T_1 (or T_2)

Total time taken by either of the projectile

Problem 2 A particle is projected with speed 10 m/s at an angle 60^0 with horizontal. Find :

(a) Time of flight
(b) Range
(c) Maximum height
(d) Velocity of particle after one second.
(e) Velocity when height of the particle is 1 m

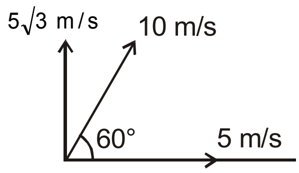
Answer :

(a) $\sqrt{3}$ sec.
(b) $5\sqrt{3}$ m
(c) $\dfrac{15}{4}$ m
(d) $10\sqrt{2-\sqrt{3}}$ m/s
(e) $\vec{v} = 5\hat{i} \pm \sqrt{55}\hat{j}$

Solution :

(a) $T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 5\sqrt{3}}{10} = \sqrt{3}$ sec.

(b) Range $= \dfrac{u^2 \sin 2\theta}{g} = \dfrac{10\times 10\times 2\times \sin 60^\circ \cos 60^\circ}{10}$



$= 0.20 \times \dfrac{\sqrt{3}}{2} \times \dfrac{1}{2} = 5\sqrt{3}$ m.

(c) maximum height $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{10\times 10\times .3}{2\times\ 10\times 4} = \dfrac{15}{4}$ m

(d) velocity at any time 't'

$\vec{v} = v_x\hat{i} + v_y$ $\Rightarrow$ $\vec{v}_x = \vec{u}_x$ $\Rightarrow$ $v_x = 5$

$\vec{v}_y = \vec{u}_y + \vec{a}_y t$ $\Rightarrow$ $v_y = 5\sqrt{3} - 10 \times 1$

$\vec{v} = 5\hat{i} + (5\sqrt{3} - 10)\hat{j}$ $\Rightarrow$ $v = 10\left(\sqrt{2-\sqrt{3}}\right)$ m/s

(e) $v^2 = u^2 + 2gh$ velocity at any height 'h' is $\vec{v} = v_x\hat{i} + v_y\hat{j}$; $v_x = u_x = 5$

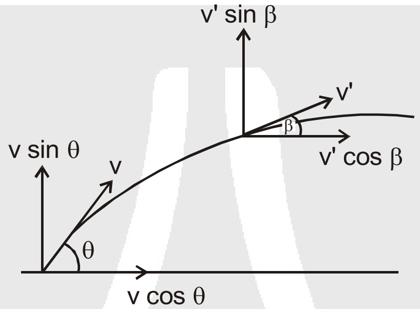
$v_y = u^2_y - 2gh = \left(5\sqrt{3}\right)^2 - 2 \times 10 \times 1$

$v_y = \sqrt{55}$ $\Rightarrow$ $\vec{v} = 5\hat{i} \pm \sqrt{55}\,\hat{j}$

Problem 3 A stone is thrown with a velocity v at angle θ with horizontal. Find its speed when it makes an angle β with the horizontal.

Answer : $\frac{v\cos\beta}{\cos\theta}$

Solution : $v\cos\theta = v'\cos\beta$



$v' = \frac{v\cos\theta}{\cos\beta}$

Problem 4 Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect the resistance of paper and air.

Answer : 700 m/s

Solution : Equation of motion in x direction $100 = v \times t$

$t = \frac{100}{v}$ ......(1)

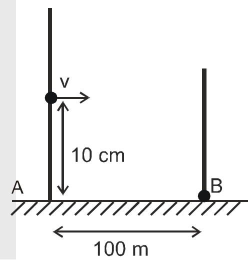
in y direction

$0.1 = 1/2 \times 9.8 \times t^2$ ......(2)

$0.1 = 1/2 \times 9.8 \times (100/v)^2$

From equation (1) & (2)

on solving we get $u = 700$ m/s



Problem 5 Two stones A and B are projected simultaneously from the top of a 100 m high tower. Stone B is projected horizontally with speed 10 m/s, and stone A is dropped from the tower. Find out the following ($g = 10$ m/s^2)

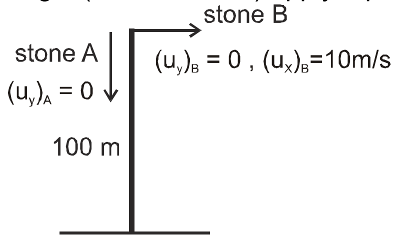
(a) Time of flight of the two stone
(b) Distance between two stones after 3 sec.
(c) Angle of strike with ground
(d) Horizontal range of particle B.

Answer :

(a) $2\sqrt{5}$ sec.
(b) $x_B = 30$ m, $y_B = 45$ m
(c) $\tan^{-1} 2\sqrt{5}$
(d) $20\sqrt{5}$ m

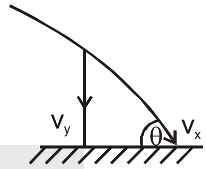
Solution :

(a) To calculate time of flight (for both stone) apply equation of motion in y direction



100 = 1/2 gt^2
t = 2 sec.

(b) $X_B = 10 \times 3 = 30$ mt
$Y_B = 1/2 \times g \times t^2 = 1/2 \times 10 \times 3 \times 3$
$Y_B = 45$ m
distance between two stones after 3 sec. $X_B = 30$,
$Y_B = 45$
So, distance = $\sqrt{(30)^2 + (45)^2}$



(c) angle of striking with ground $v_y^2 = u_y^2 + 2gh = 0 + 2 \times 10 \times 100$
$v_y = 20\sqrt{5}$ m/s $\Rightarrow$ $v_x = 10$ m/s

$\because \vec{v} = v_x\hat{i} + v_y\hat{j}$ $\Rightarrow$ $\tan\theta = \frac{v_y}{v_x}$ $\Rightarrow \theta = \tan^{-1}\left(\frac{20\sqrt{5}}{10}\right) = \tan^{-1}(2\sqrt{5})$

(d) Horizontal range of particle 'B' $X_B = 10 \times (2\sqrt{5}) = 20\sqrt{5}$ m

Problem 6 Two particles are projected simultaneously with the same speed V in the same vertical plane with angles of elevation θ and 2θ, where $\theta < 45^o$. At what time will their velocities be parallel.

Answer : $\frac{v}{g}\cos\left(\frac{\theta}{2}\right)\text{cosec}\left(\frac{3\theta}{2}\right)$

Solution : Velocity of particle projected at angle 'θ' after time t

$\vec{V}_1 = (v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}) - (gt\,\hat{j})$

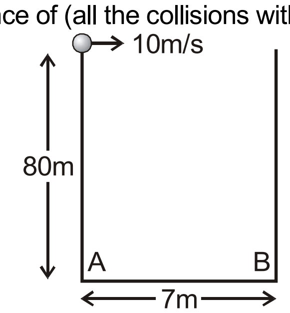
Velocity of particle projected at angle '2θ' after time t

$\vec{V}_2 = (v\cos 2\theta\,\hat{i} + v\sin 2\theta\,\hat{j}) - (gt\,\hat{j})$

Since velocities are parallel so $\frac{v_x}{v_x'} = \frac{v_y}{v_y'} \Rightarrow \frac{v\cos\theta}{v\cos 2\theta} = \frac{v\sin\theta - gt}{v\sin 2\theta - gt}$

Solving above equation we can get result. $\frac{v}{g}\cos\left(\frac{\theta}{2}\right)\text{cosec}\left(\frac{3\theta}{2}\right)$

Problem 7 A ball is projected horizontally from top of a 80 m deep well with velocity 10 m/s. Then particle will fall on the bottom at a distance of (all the collisions with the wall are elastic and wall is smooth).



(A) 5 m from A (B) 5 m from B (C) 2 m from A (D) 2 m from B

Answer : (B) 5 m from B (C) 2 m from A

Solution : Total time taken by the ball to reach at bottom = $\sqrt{\frac{2H}{g}} = \sqrt{\frac{2x80}{10}}$ = 4 sec.

Let time taken in one collision is t
Then t x 10 = 7
t = .7 sec.

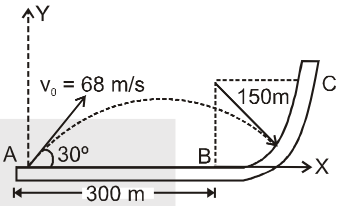
No. of collisions = $\frac{4}{.7} = 5\frac{5}{7}$ (5th collisions from wall B)

Horizontal distance travelled in between 2 successive collisions = 7 m

$\therefore$ Horizontal distance travelled in 5/7 part of collisions = $\frac{5}{7}$x7 = 5 m

Distance from A is 2 m. **Ans.**

Problem 8 A projectile is launched from point 'A' with the initial conditions shown in the figure. BC part is circular with radius 150 m. Determine the 'x' and 'y' co-ordinates of the point of impact.



Solution : Let the projectile strikes the circular path at (x, y) and 'A' to be taken as origin. From the figure co-ordinates of the centre of the circular path is (300, 150). Then the equation of the circular path is

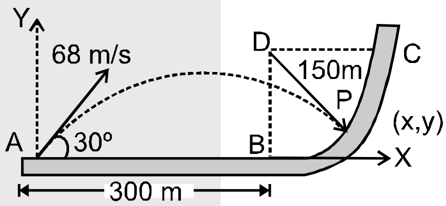
$(x – 300)^2 + (y – 150)^2 = (150)^2$ .....(1)

and the equation of the trajectory is

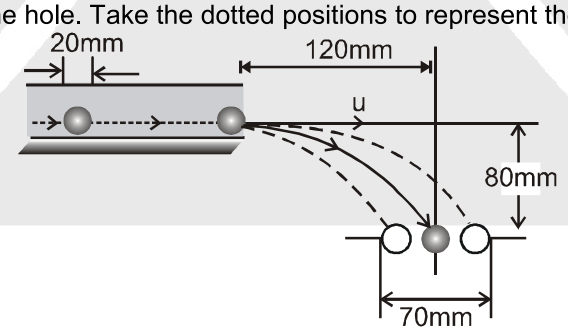
$y = x \tan 30^\circ - \frac{1}{2}\frac{gx^2}{(68)^2 \cos^2 30^\circ}$

$y = \frac{x}{\sqrt{3}} - \frac{2x^2g}{9248}$ .....(2)

From Eqs. (1) and (2) we get
x = 373 m ; y = 18.75 m



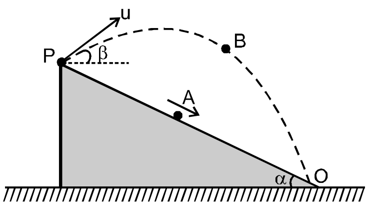
Problem 9 Ball bearings leave the horizontal through with a velocity of magnitude 'u' and fall through the 70 mm diameter hole as shown. Calculate the permissible range of 'u' which will enable the balls to enter the hole. Take the dotted positions to represent the limiting conditions.



Solution : $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 0.08}{9.8}}$ = 0.13s

$u_{min} = \frac{(120 - 35 + 10) \times 10^{-3}}{0.13}$ = 0.73 m/s and $u_{max} = \frac{(120 + 35 - 10) \times 10^{-3}}{0.13}$ = 1.11 m/s .

Problem 10 Particle A is released from a point P on a smooth inclined plane inclined at an angle α with the horizontal. At the same instant another particle B is projected with initial velocity u making an angle β with the horizontal. Both the particles meet again on the inclined plane. Find the relation between α and β.



Solution : Consider motion of B along the plane initial velocity = u cos ($\alpha + \beta$)

acceleration = g sin α

$\therefore$ OP = u cos $(\alpha + \beta)$ t + $\frac{1}{2}$g sin $(\alpha)t^2$ .....(i)

For motion of particle A along the plane,

initial velocity = 0

acceleration = g sin α

$\therefore$ OP = $\frac{1}{2}$g sin α t^2 .....(ii)

From Equation. (i) and (ii) u cos $(\alpha + \beta)$ t = 0

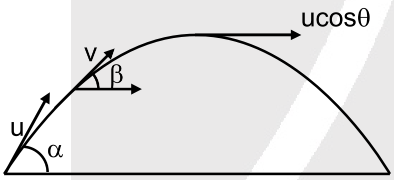
So, either t = 0 or $\alpha + \beta = \frac{\pi}{2}$

Thus, the condition for the particles to collide again is $\alpha + \beta = \pi/2$.

Problem 11 The direction of motion of a projectile at a certain instant is inclined at an angle α to the horizontal. After t seconds it is inclined an angle β. Find the horizontal component of velocity of projection in terms of g, t, α and β. (α and β are positive in anticlockwise direction)

Answer : $\frac{gt}{\tan\alpha - \tan\beta}$

Solution :



Now $a_x = 0$

$\therefore$ u cosα = v cosβ. ....(1)

Now for motion along y-axis

$a_y = -g$

$\therefore$ u sinα – gt = v sinβ ....(2)

Putting the value of v

$v = \frac{u\cos\alpha}{\cos\beta}$ in (2)

we have, u sinα – gt$=\frac{u\cos\alpha}{\cos\beta}$ sinβ.

or u sinα – u cosα tan β = gt

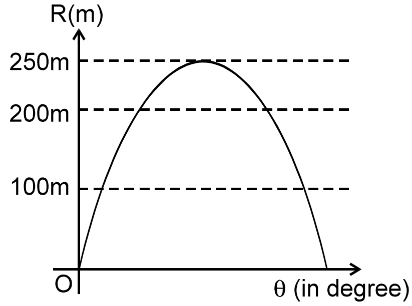
u { sinα – cosα tanβ} = gt

$u = \frac{gt}{(\sin\alpha - \cos\alpha\tan\beta)}$

Horizontal component. of velocity = u cosα.

$= \frac{gt\cos\alpha}{(\sin\alpha - \cos\alpha\tan\beta)} = \frac{gt}{(\tan\alpha - \tan\beta)}$

Problem 12 From the ground level, a ball is to be shot with a certain speed. Graph shows the range R it will have versus the launch angle θ. The least speed the ball will have during its flight if θ is chosen such that the flight time is half of its maximum possible value, is equal to (take g = 10 m/s^2)



(A) 250 m/s (B) $50\sqrt{3}$ m/s (C) 50 m/s (D) $25\sqrt{3}$ m/s

Answer : **(D)**

Solution : From the R v/s θ curve (for u = const.)

$R_{max} = \frac{u^2}{g} = 250 \quad \Rightarrow \quad u = 50$ m/sec.

T = 1/2 $T_{max.}$ possible

$$\frac{2u\sin\theta}{g} = \frac{1}{2}\left(\frac{2u}{g}\right)$$

$\Rightarrow \quad \sin\theta = 1/2 \quad \Rightarrow \quad \theta = 30°$

Least speed during flight = u cos θ = 50 cos 30 = $25\sqrt{3}$

Problem 13 The coordinates of a particle moving in a plane are given by x(t) = a cos (pt) and y(t) = b sin (pt), where a, b (< a) and p are positive constants of appropriate dimensions then -

(A) the path of the particle is an ellipse

(B) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$

(C) the acceleration of the particle is always directed towards a focus

(D) the distance travelled by the particle in time interval t = 0 to $= \pi/2p$ is a. **[JEE 1999, 3/200]**

Answer : **(AB)**

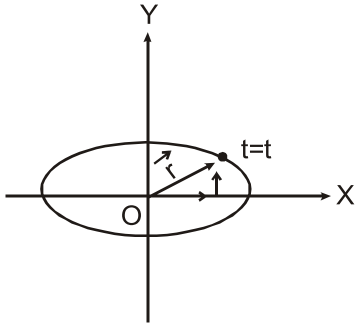
Solution : x = a cos pt $\Rightarrow$ cos (pt) = x/a ......(1)

y = b sin pt $\Rightarrow$ sin (pt) = y/b ......(2)

Squaring and adding (1) and (2), we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Therefore, path of the particle is in ellipse. Hence option (A) is correct. From the given equations we can find



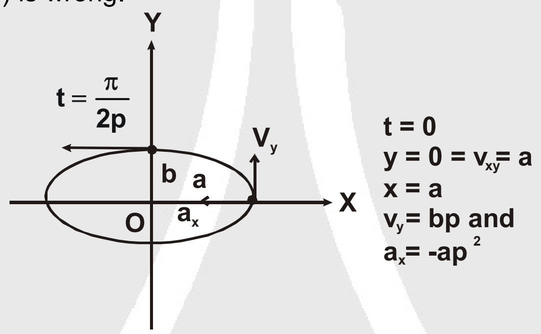
$dx/dt = v_x = -\, a\, p \sin pt$
$d^2x/dt^2 = a_x = -\, ap^2 \cos pt$
$dy/dt = v_y = bp \cos pt$
$d^2x/dt^2 = a_y = -\, bp^2 \sin pt$
At time $t = \pi/2p$ or $pt = \pi/2$
a_x and v_y become zero (become $\cos \pi/2 = 0$) only V_x and a_y are left,
or we can say that velocity is along negative x-axis and acceleration along –y-axis.
Hence at $t = \pi/2p$, velocity and acceleration of the particle are normal to each other. So option (B) is also correct.
At t = t, position of the particle

$$\vec{r}(t) = x\hat{i} + y\hat{j} = a \cos pt\, \hat{i} + b \sin pt\, \hat{j}$$

and acceleration of the particle is $\vec{a}(t) = a_x\hat{i} + a_y\hat{j}$

$= -\, p^2[a \cos pt\, \hat{i} + b \sin pt\, \hat{j}] = -\, p^2\, [\, x\, \hat{i} + y\, \hat{j}\,] = -\, p^2\, \vec{r}(t)$

Therefore acceleration of the particle is always directed towards origin.
Hence option (C) is also correct.
At t = 0, particle is at (a, 0) and at $t = \pi/2p$, particle is at (0, b). Therefore, the distance covered is one-fourth of the elliptical path not a.
Hence option (D) is wrong.



Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Definition, Projectile on a horizontal plane

A-1. Two bodies are projected at angles θ and $(90 - \theta)$ to the horizontal with the same speed. Find the ratio of their times of flight?

A-2. In above question find the ratio of the maximum vertical heights ?

A-3.✍ A body is so projected in the air that the horizontal range covered by the body is equal to the maximum vertical height attained by the body during the motion. Find the angle of projection ?

A-4. A projectile can have the same range R for two angles of projections at a given speed. If T_1 & T_2 be the times of flight in two cases, then find out relation between T_1, T_2 and R ?

A-5. A cricketer can throw a ball to a maximum horizontal distance of 100 m. To what height above the ground can the cricketer throw the same ball with same speed.

A-6. A player kicks a football at an angle of 45º with an initial speed of 20 m/s. A second player on the goal line 60 m away in the direction of kick starts running to receive the ball at that instant. Find the constant speed of the second player with which he should run to catch the ball before it hits the ground [g = 10 m/s^2]

Section (B) : Projectile from a tower

B-1. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find : (take g = 9.8 m/s^2)
(i) The time taken to reach the ground
(ii) The distance of the target from the foot of hill
(iii) The velocity with which the particle hits the ground

B-2.✍ From the top of a tower of height 50m a ball is projected upwards with a speed of 30 m/s at an angle of 30° to the horizontal. Then calculate -
(i) Maximum height from the ground
(ii) At what distance from the foot of the tower does the projectile hit the ground.
(iii) Time of flight.

Section (C) : Equation of trajectory

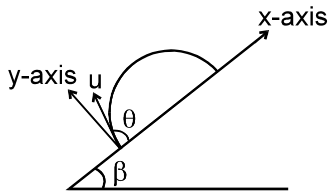
C-1.✍ The equation of a projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$, find the angle of projection. Also find the speed of projection. Where at t = 0, x = 0 and y = 0 also $\frac{d^2x}{dt^2} = 0$ & $\frac{d^2y}{dt^2} = -g$.

C-2 A bullet is fired from horizontal ground at some angle passes through the point $\left(\frac{3R}{4}, \frac{R}{4}\right)$, where 'R' is the range of the bullet. Assume point of the fire to be origin and the bullet moves in x-y plane with x-axis horizontal and y-axis vertically upwards. Angle of projection is $\frac{\alpha\pi}{180}$ radian. Find α :

C-3. The radius vector of a point A relative to the origin varies with time t as $\vec{r} = at\hat{i} - bt^2\hat{j}$, where a and b are positive constants and $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find:
(i) The equation of the point's trajectory y(x) ; plot this function
(ii) The time dependence of the velocity **v** and acceleration **a** vectors as well as of the moduli of these quantities.

Section (D) : Projectile on an inclined plane

D-1. A particle is projected at an angle θ with an inclined plane making an angle β with the horizontal as shown in figure, speed of the particle is u, after time t find :



(a) x component of acceleration ?
(b) y component of acceleration ?
(c) x component of velocity ?
(d) y component of velocity ?
(e) x component of displacement ?
(f) y component of displacement ?
(g) y component of velocity when particle is at maximum distance from the incline plane ?

PART - II : ONLY ONE OPTIONS CORRECT TYPE

Section (A) : Definition, Projectile on a horizontal plane

A-1. A ball is thrown upwards. It returns to ground describing a parabolic path. Which of the following remains constant?
(A) Speed of the ball (B) Kinetic energy of the ball
(C) Vertical component of velocity (D) Horizontal component of velocity.

A-2. A bullet is fired horizontally from a rifle at a distant target. Ignoring the effect of air resistance, which of the following is correct?

	Horizontal Acceleration	Vertical Acceleration
(A)	$10\ ms^{-2}$	$10\ ms^{-2}$
(B)	$10\ ms^{-2}$	$0\ ms^{-2}$
(C)	$0\ ms^{-2}$	$10\ ms^{-2}$
(D)	$0\ ms^{-2}$	$0\ ms^{-2}$.

A-3. A point mass is projected, making an acute angle with the horizontal. If angle between velocity and acceleration $\vec{g}$ is θ at any time t during the motion, then θ is given by
(A) $0^\circ < \theta < 90^\circ$ (B) $\theta = 90^\circ$ (C) $\theta < 90^\circ$ (D) $0^\circ < \theta < 180^\circ$

A-4. A projectile is thrown with a speed v at an angle θ with the upward vertical. Its average velocity between the instants at which it crosses half the maximum height is
(A) v sin θ, horizontal and in the plane of projection
(B) v cos θ, horizontal and in the plane of projection
(C) 2v sin θ, horizontal and perpendicular to the plane of projection
(D) 2v cos θ, vertical and in the plane of projection.

A-5. A particle moves along the parabolic path $y = ax^2$ in such a way that the x component of the velocity remains constant, say c. The acceleration of the particle is
(A) $ac\ \hat{k}$ (B) $2ac^2\ \hat{j}$ (C) $ac^2\ \hat{j}$ (D) $a^2c\ \hat{j}$

A-6. During projectile motion, acceleration of a particle at the highest point of its trajectory is
(A) g (B) zero
(C) less than g (D) dependent upon projection velocity

A-7. The speed at the maximum height of a projectile is half of its initial speed u. Its range on the horizontal plane is:
(A) $\frac{2u^2}{3g}$ (B) $\frac{\sqrt{3}u^2}{2g}$ (C) $\frac{u^2}{3g}$ (D) $\frac{u^2}{2g}$

A-8. The velocity of projection of a projectile is $(6\hat{i} + 8\hat{j})$ ms^{-1}. The horizontal range of the projectile is (g = 10 m/sec^2)

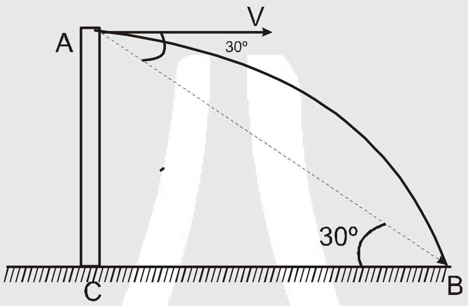
(A) 4.9 m (B) 9.6 m (C) 19.6 m (D) 14 m

Section (B) : Projectile from a tower

B-1.* One stone is projected horizontally from a 20 m high cliff with an initial speed of 10 ms^{-1}. A second stone is simultaneously dropped from that cliff. Which of the following is true?

(A) Both strike the ground with the same speed.
(B) The stone with initial speed 10 ms^{-1} reaches the ground first.
(C) Both the stones hit the ground at the same time.
(D) The stone which is dropped from the cliff reaches the ground first.

B-2. An object is thrown horizontally from a point 'A' from a tower and hits the ground 3s later at B. The line from 'A' to 'B' makes an angle of 30º with the horizontal. The initial velocity of the object is : (take g = 10 m/s^2)

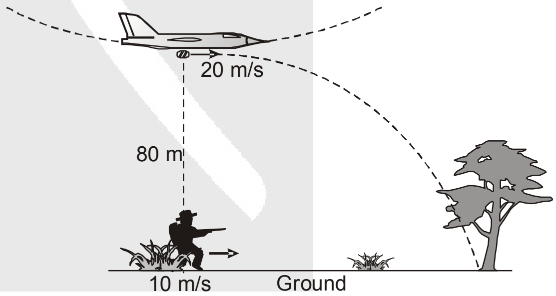


(A) $15\sqrt{3}$ m/s (B) 15 m/s (C) $10\sqrt{3}$ m/s (D) $25/\sqrt{3}$ m/s

B-3. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at angle 45º. What is the vertical component of velocity when it strikes the ground?

(A) $18\sqrt{3}$ ms^{-1} (B) 18 ms^{-1} (C) $9\sqrt{2}$ ms^{-1} (D) 9 ms^{-1}

B-4. A bomber plane moving at a horizontal speed of 20 m/s releases a bomb at a height of 80 m above ground as shown. At the same instant a Hunter of negligible height starts running from a point below it, to catch the bomb with speed 10 m/s. After two seconds he realized that he cannot make it, he stops running and immediately holds his gun and fires in such direction so that just before bomb hits the ground, bullet will hit it. What should be the firing speed of bullet.(Take g = 10 m/s^2)



(A) 10 m/s (B) $20\sqrt{10}$ m/s (C) $10\sqrt{10}$ m/s (D) None of these

Section (C) : Equation of trajectory

C-1. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y varies with time t in second as:

$$x = 10\sqrt{3}t \text{ and } y = 10t - t^2$$

The maximum height attained by the ball is

(A) 100 m (B) 75 m (C) 50 m (D) 25 m.

Section (D) : Projectile on an inclined plane

D-1. A plane surface is inclined making an angle θ with the horizontal. From the bottom of this inclined plane, a bullet is fired with velocity v. The maximum possible range of the bullet on the inclined plane is

(A) $\frac{v^2}{g}$ (B) $\frac{v^2}{g(1+\sin\theta)}$ (C) $\frac{v^2}{g(1-\sin\theta)}$ (D) $\frac{v^2}{g(1+\cos\theta)}$

D-2. A ball is horizontally projected with a speed v from the top of a plane inclined at an angle 45º with the horizontal. How far from the point of projection will the ball strike the plane?

(A) $\frac{v^2}{g}$ (B) $\frac{\sqrt{2}v^2}{g}$ (C) $\frac{2v^2}{g}$ (D) $\left[\frac{2\sqrt{2}v^2}{g}\right]$

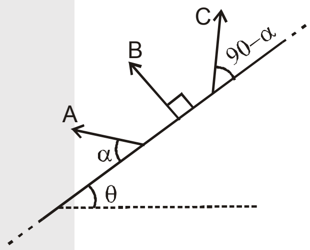
D-3. A particle is projected at angle 37° with the incline plane in upward direction with speed 10 m/s. The angle of incline plane is given 53°. Then the maximum distance from the incline plane attained by the particle will be -

(A) 3m (B) 4 m (C) 5 m (D) zero

D-4. On an inclined plane of inclination 30º, a ball is thrown at an angle of 60º with the horizontal from the foot of the incline with a velocity of $10\sqrt{3}$ ms^{-1}. If g = 10 ms^{-2}, then the time in which ball will hit the inclined plane is -

(A) 1 sec. (B) 6 sec. (C) 2 sec. (D) 4 sec.

D-5. Three stones A, B, C are projected from surface of very long inclined plane with equal speeds and different angles of projection as shown in figure. The incline makes an angle θ with horizontal. If H_A, H_B and H_C are maximum height attained by A, B and C respectively above inclined plane then: (Neglect air friction)

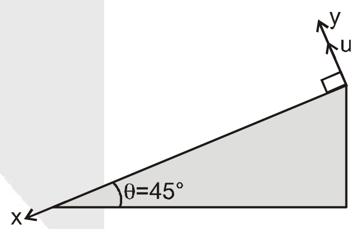


(A) $H_A + H_C = H_B$ (B) $H_A^2 + H_C^2 = H_B^2$

(C) $H_A + H_C = 2H_B$ (D) $H_A^2 + H_C^2 = 2H_B^2$

PART - III : MATCH THE COLUMN

1. An inclined plane makes an angle θ = 45° with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at t = 0 sec. x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in column I with the results in column II. (g in column II is acceleration due to gravity.)



Column I	Column II
(A) The instant of time at which velocity of stone is parallel to x-axis	(p) $\frac{2\sqrt{2}u}{g}$
(B) The instant of time at which velocity of stone makes an angle θ = 45° with positive x-axis. in clockwise direction	(q) $\frac{2u}{g}$
(C) The instant of time till which (starting from t = 0) component of displacement along x-axis become half the range on inclined plane is	(r) $\frac{\sqrt{2}u}{g}$
(D) Time of flight on inclined plane is	(s) $\frac{u}{\sqrt{2}g}$

2. A particle is projected from level ground. Assuming projection point as origin, x-axis along horizontal and y-axis along vertically upwards. If particle moves in x-y plane and its path is given by $y = ax - bx^2$ where a, b are positive constants. Then match the physical quantities given in column-I with the values given in column-II. (g in column II is acceleration due to gravity.)

Column I	Column II
(A) Horizontal component of velocity	(p) a/b
(B) Time of flight	(q) $\frac{a^2}{4b}$
(C) Maximum height	(r) $\sqrt{\frac{g}{2b}}$
(D) Horizontal range	(s) $\sqrt{\frac{2a^2}{bg}}$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. A particle moves in the xy plane with only an x-component of acceleration of 2 ms^{-2}. The particle starts from the origin at t = 0 with an initial velocity having an x-component of 8 ms^{-1} and y-component of –15 ms^{-1}. Velocity of particle after time t is :

(A) $[(8 + 2t)\hat{i} - 15\hat{j}]$ m s^{-1} (B) zero
(C) $2t\hat{i} + 15\hat{j}$ (D) directed along z-axis.

2. A plane flying horizontally at a height of 1500 m with a velocity of 200 ms^{-1} passes directly overhead an antiaircraft gun. Then the angle with the horizontal at which the gun should be fired for the shell with a muzzle velocity of 400 m s^{-1} to hit the plane, is -

(A) 90º (B) 60º (C) 30º (D) 45º

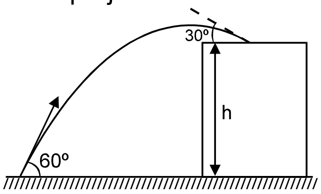
3. If R and h represent the horizontal range and maximum height respectively of an oblique projection whose start point (i.e. point of projecteion) & end point are in same horizontal level. Then $\frac{R^2}{8h} + 2h$ represents

(A) maximum horizontal range (B) maximum vertical range
(C) time of flight (D) velocity of projectile at highest point

4. A projectile is thrown with velocity v making an angle θ with the horizontal. It just crosses the top of two poles, each of height h, after 1 second and 3 second respectively. The time of flight of the projectile is

(A) 1 s (B) 3 s (C) 4 s (D) 7.8 s.

5. A stone projected at an angle of 60º from the ground level strikes at an angle of 30º on the roof of a building of height 'h'. Then the speed of projection of the stone is :

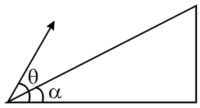


(A) $\sqrt{2gh}$ (B) $\sqrt{6gh}$ (C) $\sqrt{3gh}$ (D) $\sqrt{gh}$

6. A particle at a height 'h' from the ground is projected with an angle 30° from the horizontal, it strikes the ground making angle 45° with horizontal. It is again projected from the same point at height h with the same speed but with an angle of 60° with horizontal. Find the angle it makes with the horizontal when it strikes the ground :

(A) $\tan^{-1}(4)$ (B) $\tan^{-1}(5)$ (C) $\tan^{-1}(\sqrt{5})$ (D) $\tan^{-1}(\sqrt{3})$

7. A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane of inclination α as shown in figure.



(A) $\sin\alpha = \cos(\theta - \alpha)$ (B) $\cos\alpha = \sin(\theta - \alpha)$ (C) $\tan\theta = \cot(\theta - \alpha)$ (D) $\cot(\theta - \alpha) = 2\tan\alpha$

8. A ball is thrown eastward across level ground. A wind blows horizontally to the east, and assume that the effect of wind is to provide a constant force towards the east, equal in magnitude to the weight of the ball. The angle θ (with horizontal east) at which the ball should be projected so that it travels maximum horizontal distance is
(A) 45° (B) 37° (C) 53° (D) 67.5°

PART - II : NUMERICAL VALUE

1. A hunter at the bottom of a slant hill is trying to shoot a deer on a hill. The distance of the deer along his line of sight is $10\sqrt{181}$ meters and the height of the hill is 90 meters. His gun has a muzzle velocity of 100 m/sec. Minimum how many meters above the deer should he aim his rifle in order to hit it? [$g = 10\ m/s^2$]

2. A stone is thrown in such a manner that it would just hit a bird at the top of a tree and afterwards reach a maximum height double that of the tree. If at the moment of throwing the stone the bird flies away horizontally with constant velocity and the stone hits the bird after some time.The ratio of horizontal velocity of stone to that of the bird is $\frac{1}{n} + \frac{1}{\sqrt{n}}$. Find 2n.

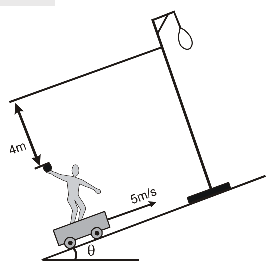
3. If 4 seconds be the time in which a projectile reaches a point P of its path and 5 seconds the time from P till it reaches the horizontal plane passing through the point of projection. The height of P above the horizontal plane (in m) will be - [$g = 9.8\ m/sec^2$]

4. A person standing on the top of a cliff 30 m high has to throw a packet to his friend standing on the ground 40 m horizontally away. If he throws the packet directly aiming at the friend with a speed of $\frac{125}{3}$ m/s. Packet falls at a distance $\frac{20}{\alpha}$ m from the friend. Here α is an integer. Find α. [Use $g = 10\ m/s^2$].

5. A particle is projected from a point (0, 1) on Y-axis (assume + Y direction vertically upwards) aiming towards a point (4, 9). It falls on ground on x axis in 1 sec. If the speed of projection is $\sqrt{\beta}$ m/s, where β is an integer. Find β**.** Taking $g = 10\ m/s^2$ and all coordinate in metres.

6. A Bomber flying upward at an angle of 53º with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 sec after its release. Velocity of the bomber at the time of release of the bomb is V m/s. Find V/4. [Given $\sin 53º = 0.8$; $g = 10\ ms^{-2}$]

7. A man is travelling on a flat car which is moving up a plane inclined at $\cos\theta = 4/5$ to the horizontal with a speed 5 m/s. He throws a ball towards a stationary hoop located perpendicular to the incline in such a way that the ball moves parallel to the slope of the incline while going through the centre of the hoop. The centre of the hoop is 4 m high from the man's hand calculate the time taken by the ball to reach the hoop in second.



8. A stone is projected horizontally with speed v from a height h above ground. A horizontal wind is blowing in direction opposite to velocity of projection and gives the stone a constant horizontal acceleration f (in direction opposite to initial velocity). As a result the stone falls on ground at a point vertically below the point of projection. Then find the value of $\frac{f^2h}{gv^2}$ (g is acceleration due to gravity)

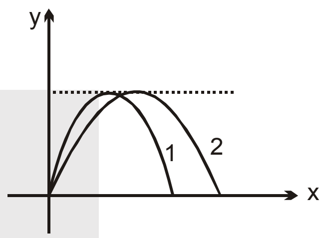
9. If at an instant the velocity of a projectile be 60 m/s and its inclination to the horizontal be 30°, at what time interval (in sec) after that instant will the particle be moving at right angles to its former direction. ($g = 10\ m/s^2$)

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A projectile is projected at an angle α (> 45°) with an initial velocity u. The time t at which its horizontal component will equal the vertical component in magnitude:
(A) $t = u/g\ (\cos\alpha - \sin\alpha)$
(B) $t = u/g\ (\cos\alpha + \sin\alpha)$
(C) $t = u/g\ (\sin\alpha - \cos\alpha)$
(D) $t = u/g\ (\sin^2\alpha - \cos^2\alpha)$

2. At what angle should a body be projected with a velocity $24\ ms^{-1}$ just to pass over the obstacle 14 m high at a distance of 24 m. [Take $g = 10\ ms^{-2}$]
(A) $\tan\theta = 19/5$ (B) $\tan\theta = 1$ (C) $\tan\theta = 3$ (D) $\tan\theta = 2$

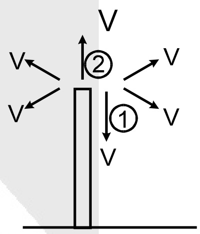
3. Two stones are projected from level ground. Trajectories of two stones are shown in figure. Both stones have same maximum heights above level ground as shown. Let T_1 and T_2 be their time of flights and u_1 and u_2 be their speeds of projection respectively (neglect air resistance). Then



(A) $T_2 > T_1$
(B) $T_1 = T_2$
(C) $u_1 > u_2$
(D) $u_1 < u_2$

4. A projectile of mass 1 kg is projected with a velocity of $\sqrt{20}$ m/s such that it strikes on the same level as the point of projection at a distance of $\sqrt{3}$ m. Which of the following options are correct ?
(A) The maximum height reached by the projectile can be 0.25 m.
(B) The minimum velocity during its motion can be $\sqrt{15}$ m/s.
(C) The time taken for the flight can be $\sqrt{\frac{3}{5}}$ s.
(D) Maximum angle of projection can be 60°.

5. Particles are projected from the top of a tower with same speed at different angles as shown. Which of the following are True ?

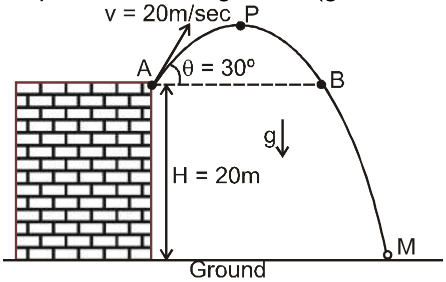


(A) All the particles would strike the ground with (same) speed.
(B) All the particles would strike the ground with (same) speed simultaneously.
(C) Particle 1 will be the first to strike the ground.
(D) Particle 1 strikes the ground with maximum speed.

PART - IV : COMPREHENSION

Comprehension-1

A ball is projected with initial velocity u = 20 m/sec at an angle θ = 30º (from horizontal) from point A which is at a height H = 20m above horizontal. P is the highest point for complete motion of particle, whereas M is the point at which particle strikes ground. ($g = 10\ m/s^2$)



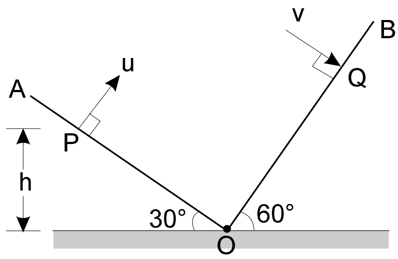
1. Velocity (along vertical direction) of the particle at point P is :
(A) 0 m/sec (B) $10\sqrt{3}$ m/sec (C) $5\sqrt{3}$ m/sec (D) $4\sqrt{3}$ m/sec

2. Total time of flight (from A to M) of the projectile is :

(A) 2 sec (B) $(\sqrt{5}+1)$ sec (C) $(\sqrt{5}-1)$ (D) $(2+\sqrt{5})$ sec

Comprehension-2✍

Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}\ \frac{m}{s}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q (Take g = 10 m/s^2). Then



3.✍ The time of flight from P to Q is :-

(A) 5 Sec. (B) 2 sec (C) 1 sec (D) None of these

4.✍ The speed with which the particle strikes the plane OB is :

(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

5.✍ The height h of point P from the ground is :-

(A) $10\sqrt{3}$ m (B) 10 m (C) 5 m (D) 20 m

6.✍ The distance PQ is :

(A) 20 m (B) $10\sqrt{3}$ m (C) 10 m (D) 5 m

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

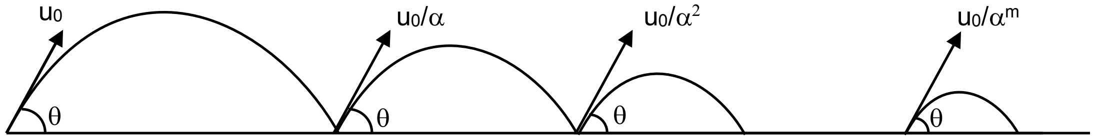
1. Shots fired simultaneously from the top and foot of a vertical cliff at elevations of 30° and 60° respectively, strike an object simultaneously which is at a height of 100 meters from the ground and at a horizontal distance of $200\sqrt{3}$ meters from the cliff. Find the height of the cliff, the velocities of projection of the shots and the time taken by the shots to hit the object. (g = 10 m/sec^2.) **[REE 2000; 5/100]**

2. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is ___________.

[JEE (Advanced) 2018; 3/60]

3. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is __________

[JEE (Advanced) 2019; 3/62]



PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is : **[AIEEE 2009; 4/144]**

(1) $7\sqrt{2}$ units (2) 7 units (3) 8.5 units (4) 10 units

2. A particle is moving with velocity $\vec{v}=K(y\hat{i}+x\hat{j})$, where K is a constant. The general equation for its path is: **[AIEEE 2010; 4/144]**

(1) $y = x^2$ + constant (2) $y^2 = x$ + constant (3) xy = constant (4) $y^2 = x^2$ + constant

3. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is : **[AIEEE 2011; 4/120, –1]**

(1) $\pi\frac{v^2}{g}$ (2) $\pi\frac{v^4}{g^2}$ (3) $\frac{\pi}{2}\frac{v^4}{g^2}$ (4) $\pi\frac{v^2}{g^2}$

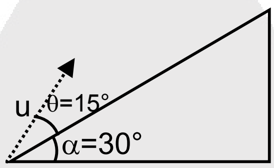
4. A boy can throw a stone up to a maximum height of 10m. The maximum horizontal distance that the boy can throw the same stone up to will be : **[AIEEE 2012 ; 4/120, –1]**

(1) $20\sqrt{2}$m (2) 10 m (3) $10\sqrt{2}$m (4) 20m

5. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is : **[JEE (Main) 2013; 4/120]**

(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$ (3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

6. A plane is inclined at an angle α = 30° with respect to the horizontal. A particle is projected with a speed u = 2 ms^{-1} from the base of the plane making an angle θ = 15° with respect to the plane as shown in the figure. The distance from the base at which the particle hits the plane is close to : (Take g = 10 ms^{-2}) **[JEE (Main) 2019; 4/120, –1]**



(1) 18 cm (2) 14 cm (3) 26 cm (4) 20 cm

Answers

EXERCISE-1

PART - I

Section (A)

A-1. $\tan\theta : 1$ **A-2.** $\tan^2\theta : 1$

A-3. $\tan\theta = 4$ or $\theta = \tan^{-1}(4)$

A-4. $T_1T_2 = 2R/g$ **A-5.** 50 m

A-6. $5\sqrt{2}$ m/s

Section (B)

B-1. (i) 10 sec. (ii) 980 m (iii) $98\sqrt{2}$ m/s

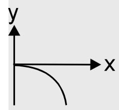
B-2. (i) 61.25 m (ii) $75\sqrt{3}$ m $\approx$ 130 m

(iii) 5 sec.

Section (C)

C-1. $\theta = 60°$, 2 m/s **C-2** 53

C-3. (i) $y = -\dfrac{bx^2}{a^2}$



(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$

acceleration $= -2b\hat{j}$,

$\left|\vec{v}\right| = \sqrt{a^2 + 4b^2t^2}$, |acceleration| = 2 b

Section (D)

D-1. (a) $-g\sin\beta$, (b) $-g\cos\beta$,

(c) $u\cos\theta - g\sin\beta \times t$,

(d) $u\sin\theta - g\cos\beta \times t$,

(e) $u\cos\theta \times t - \dfrac{1}{2}g\sin\beta \times t^2$,

(f) $u\sin\theta \times t - \dfrac{1}{2}g\cos\beta \times t^2$, (g) zero.

PART - II

Section (A)

A-1. (D) **A-2.** (C) **A-3.** (D)
A-4. (A) **A-5.** (B) **A-6.** (A)
A-7. (B) **A-8.** (B)

Section (B)

B-1. (C) **B-2.** (A) **B-3.** (B)
B-4. (C)

Section (C)

C-1. (D)

Section (D)

D-1. (B) **D-2.** (D) **D-3.** (A)
D-4. (C) **D-5.** (A)

PART - III

1. (A) r (B) s (C) q (D) p
2. (A) r ; (B) s ; (C) q ; (D) p

EXERCISE-2

PART - I

1. (A) **2.** (B) **3.** (A)
4. (C) **5.** (C) **6.** (C)
7. (D) **8.** (D)

PART - II

1. 10 **2.** 4 **3.** 98
4. 3 **5.** 20 **6.** 25
7. 1 **8.** 2 **9.** 12

PART - III

1. (BC) **2.** (AB) **3.** (BD)
4. (ABCD) **5.** (AC)

PART - IV

1. (A) **2.** (B) **3.** (B)
4. (A) **5.** (C) **6.** (A)

EXERCISE-3

PART - I

1. 400 m, $V_T = 40$ m/s ,
$V_F = 40\sqrt{3}$ m/s, T = 10 s.
2. 30 m **3.** 4

PART - II

1. (1) **2.** (4) **3.** (2)
4. (4) **5.** (2) **6.** (4)

High Level Problems (HLP)

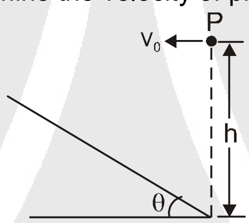
SUBJECTIVE QUESTIONS

1. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a cm short of it when the elevation is α and goes b cm too far when the elevation is β. Show that if the velocity of projection is same in all the case, the proper elevation is $\frac{1}{2}\sin^{-1}\left[\frac{b\sin 2\alpha + a\sin 2\beta}{a+b}\right]$

2. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection prove that $\tan\theta = \tan\alpha + \tan\beta$.

3. A shell bursts on contact with the ground and pieces from it fly in all directions with all velocities upto 80 feet per second. Show that a man 100 feet away is in danger for $\frac{5}{\sqrt{2}}$ seconds. [Use g = 32 ft/s^2].

4. A stone is projected horizontally from a point P, so that it hits the inclined plane perpendicularly. The inclination of the plane with the horizontal is θ and the point P is at a height h above the foot of the incline, as shown in the figure. Determine the velocity of projection.



5. Two parallel straight lines are inclined to the horizontal at an angle α. A particle is projected from a point mid way between them so as to graze one of the lines and strikes the other at right angles. Show that if θ is the angle between the direction of projection and either of lines, then

$\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$

6. The benches of a gallery in a cricket stadium are 1 m high and 1 m wide. A batsman strikes the ball at a level 1 m above the ground and hits a ball. The ball starts at 35 m/s at an angle of 53° with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110 m from the batsman. On which bench will the ball hit.

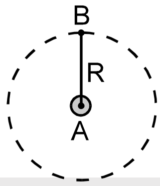
7. A ship is approaching a cliff of height 105 m above sea level. A gun fitted on the ship can fire shots with a speed of 110 ms^{-1}. Find the maximum distance from the foot of the cliff from where the gun can hit an object on the top of the cliff. [g = 10 m/s^2] **[REE 1994, 6]**

8. Shots fired simultaneously from the top and bottom of a vertical cliff with the elevation α and β respectively, strike an object simultaneously at the same point on the ground. If s is the horizontal distance of the object from the cliff, then what is the height of the cliff. ($\beta > \alpha$)

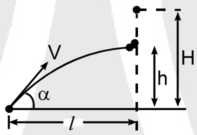
9. Some students are playing cricket on the roof of a building of height 20 m. While playing, ball falls on the ground. A person on the ground returns their ball with the minimum possible speed at angle of projection 45° with the horizontal. The speed of projection is $20\sqrt{\alpha}$ m/s. Here α is an integer. Find α

10. A cannon fires successively two shells with velocity v_0= 250 m/s; the first at the angle θ_1 = 53° and the second at the angle θ_2 = 37° to the horizontal, in the same vertical plane, neglecting the air drag, find the time interval (in sec) between firings leading to the collision of the shells. (g = 10 m/s^2).

11. A small cannon 'A' is mounted on a platform (which can be rotated so that the cannon can aim at any point) and is adjusted for maximum range R of shells. It can throw shells on any point on the shown circle (dotted) on ground. Suddenly a windstorm starts blowing in horizontal direction normal to AB with a speed $\sqrt{2}$ times the velocity of shell. At what least distance can the shell land from point B. (Assume that the velocity of the windstorm is imparted to the shell in addition to its velocity of projection. Also assume that the platform is kept stationary while projecting the shell.)



12. A body A falls freely from some altitude H (<< Re). At the moment the first body starts falling another body B is thrown from the earth's surface which collides with the first at an altitude h = H/2 . The horizontal distance of that point of collision is ℓ from the starting point of B. Find the initial velocity and the angle at which it was thrown ?



13. Two guns situated on the top of a hill of height 10m fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontaly and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at point P. Find : **[JEE 1996, 5]**
(a) The time interval between the firings and
(b) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane

14. A small ball rolls of the top of a stairway horizontally with a velocity of 4.5 m s^{-1}. Each step is 0.2 m high and 0.3 m wide. If g is 10 ms^{-2}, then the ball will strike the nth step where n is equal to (assume ball strike at the edge of the step).

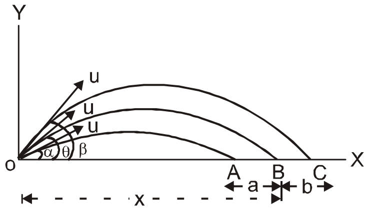
15. A particle moves along the parabolic path x = y^2 + 2y + 2 in such a way that the y-component of velocity vector remains 5m/s during the motion. The magnitude of the acceleration of the particle (in m/s^2) is :

16. A building 4.8 m high 2b meters wide has a flat roof. A ball is projected from a point on the horizontal ground 14.4 m away from the building along its width. If projected with velocity 16 m/s at an angle of 45° with the ground, the ball hits the roof in the middle, find the width 2b. Also find the angle of projection so that the ball just crosses the roof if projected with velocity $10\sqrt{3}$ m/s.(g=10m/s^2)

17. A vertical pole has a red mark at some height. A stone is projected from a fixed point on the ground. When projected at an angle of 45° it hits the pole orthogonally 1 m above the mark. When projected with a different speed at an angle of tan^{-1}(3/4), it hits the pole orthogonally 1.5 m below the mark. Find the speed and angle of projection so that it hits the mark orthogonally to the pole. [g = 10 m/sec^2] **[REE 1996, 6]**

Answers

1.



$OA = x - a = \dfrac{u^2 \sin 2\alpha}{g}$ ...(1)

$OC = x + b = \dfrac{u^2 \sin 2\beta}{g}$ ...(2)

$OB = x = \dfrac{u^2 \sin 2\theta}{g}$ ...(3)

From eqs. (1) and (2)

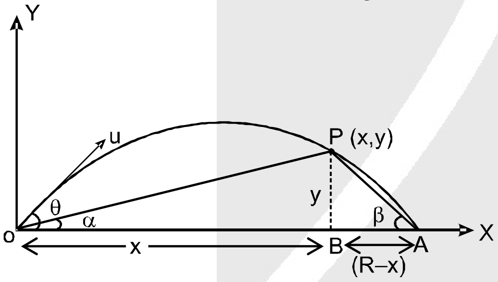
$x\,(b + a) = \left(\dfrac{b \sin 2\alpha + a \sin 2\beta}{g}\right) u^2$

Substituting the value of x from eq. (3), we get

$\dfrac{u^2 \sin 2\theta}{g}\,(b + a) = \left(\dfrac{b \sin 2\alpha + a \sin 2\beta}{g}\right) u^2$

Solving this equation, we will get θ.

2. The situation is shown in the fig.



From fig $\tan \alpha + \tan \beta = \dfrac{y}{x} + \dfrac{y}{(R - x)}$

where R is the range.

$\therefore \tan \alpha + \tan \beta = \dfrac{y(R - x) + xy}{x(R - x)}$

or $\tan \alpha + \tan \beta = \dfrac{y}{x} \times \dfrac{R}{(R - x)}$ ....(1)

but $y = x \tan \theta \left(1 - \dfrac{x}{R}\right)$

or $\tan \theta = \dfrac{y}{x} \times \dfrac{R}{(R - x)}$ ....(2)

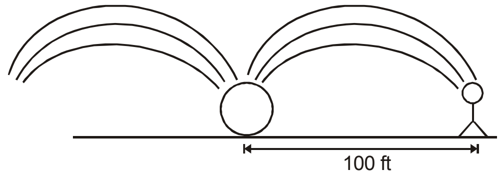
From equations (1) and (2), we have

$\tan \theta = \tan \alpha + \tan \beta$.

3. According to given problem u = 80 f / s

Range = $\dfrac{u^2 \sin 2\theta}{g}$

$\sin 2\theta = \dfrac{100 \times 32}{(80)^2} = 1/2$



$\theta = 15°$ For same Range $\theta = 15°, 75°$

Thus there will be two time of flight

$T_1 = \dfrac{2u \sin 15°}{g} = \dfrac{2 \times 80 \times \sin 15°}{32}$ (minimum time)

$\sin 15° = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$

$T_2 = \dfrac{2u \sin 75°}{g} = \dfrac{2 \times 8 \times \sin 75°}{32}$ (maximum time)

$\sin 75° = \dfrac{\sqrt{3} + 1}{2\sqrt{2}}$

Danger time = Maximum time – Minimum time = $(T_2 - T_1)$

$= \dfrac{2 \times 80}{32}\,[\sin 75° - \sin 15°]$

$= \dfrac{2 \times 80}{32}\left[\dfrac{\sqrt{3} + 1}{2\sqrt{2}} - \dfrac{\sqrt{3} - 1}{2\sqrt{2}}\right] = \dfrac{5}{\sqrt{2}}$ sec.

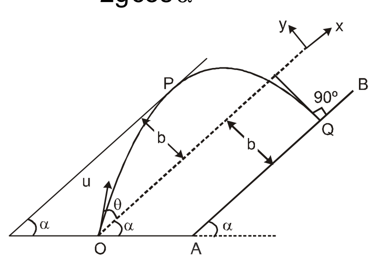
4. $v_0 = \sqrt{\dfrac{2gh}{2 + \cot^2 \theta}}$

5. Consider the motion of the particle from O to P. The velocity v_y at P is zero.

$$v_y^2 = u_y^2 + 2a_y s_y$$

$\therefore\ 0 = (u \sin \theta)^2 - 2\,(g \cos \alpha)\, b$

or $b = \dfrac{u^2 \sin^2 \theta}{2g \cos \alpha}$ ....(i)



Now, consider the motion of the particle from O to Q.

The particle strikes the point Q at 90º to AB, i.e., its velocity along x-direction is zero.

Using $v_x = u_x + a_x t$, we have

$0 = u\cos\theta - (g\sin\alpha)t$

or $t = \dfrac{u\cos\theta}{g\sin\alpha}$ ...(ii)

For motion in y-direction, $s_y = u_y t + \dfrac{1}{2}a_y t^2$

or $-b = u\sin\theta$

$\left(\dfrac{u\cos\theta}{g\sin\alpha}\right) + \dfrac{1}{2}(-g\cos\alpha)\left(\dfrac{u\cos\theta}{g\sin\alpha}\right)^2$ ...(iii)

From Eqs. (i) and (iii)

or– $\dfrac{u^2\sin^2\theta}{2g\cos\alpha} = \dfrac{u^2\sin\theta\cos\theta}{g\sin\alpha} - \dfrac{gu^2\cos\alpha\cos^2\theta}{2g^2\sin^2\alpha}$

or– $\dfrac{\sin^2\theta}{2\cos\alpha} = \dfrac{\sin\theta\cos\theta}{\sin\alpha} - \dfrac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$

Solving, we get $\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$

6. 6th step **7.** 1100 m

8. $h = s(\tan\beta - \tan\alpha)$. **9.** 2

10. 10sec

11. Let the speed of shell be u and the speed of wind be v.

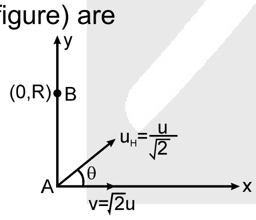
The time of flight T remains unchanged due to windstorm

$T = \dfrac{\sqrt{2}u}{g}$ ....(1)

Horizontal component of velocity of shell in absence of air

$u_H = \dfrac{u}{\sqrt{2}}$ ....(2)

Hence the net x and y component of velocity of shell (see figure) are



$u_x = \sqrt{2}u + \dfrac{u}{\sqrt{2}}\cos\theta$ ....(3a)

$u_y = \dfrac{u}{\sqrt{2}}\sin\theta$ ....(3b)

∴ The x and y coordinate of point P where shell lands is

$x = u_xT = \left(\sqrt{2}u + \dfrac{u}{\sqrt{2}}\cos\theta\right)\dfrac{\sqrt{2}u}{g} = 2R + R\cos\theta$ ....(4a)

$y = u_yT = \left(\dfrac{u}{\sqrt{2}}\sin\theta\right)\dfrac{\sqrt{2}u}{g} = R\sin\theta$ ....(4b)

∴ The distance S between B and P is given by

$S^2 = (x-0)^2 + (y-R)^2 = (2R + R\cos\theta)^2 + (R\sin\theta - R)^2$

$= R^2[6 + 4\cos\theta - 2\sin\theta]$

$= R^2\left[6 + \sqrt{20}\left(\dfrac{4\cos\theta}{\sqrt{20}} - \dfrac{2\sin\theta}{\sqrt{20}}\right)\right]$

∴ $S_{minimum} = R\sqrt{6-\sqrt{20}}$

$= R\sqrt{6-2\sqrt{5}}$ or $R(\sqrt{5}-1)$ **Ans.**

Alternate : Circle in fig. (1) Represents locus of all points where shell lands on the ground in absence of windstorm.

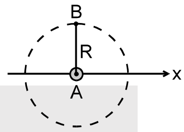


fig.1

Let the speed of shell be 'u' and the speed of wind be $v = \sqrt{2}u$. Let T be the time of flight, which remains unaltered even when the windstorm blows. Since R is the maximum range angle of projection is 45º with the horizontal.

Then $R = \dfrac{u}{\sqrt{2}}T$ .... (1)

As a result of flow of wind along x-axis, there is an additional shift (Δx) of the shell along x-axis in time of flight.

$\Delta x = vT = \sqrt{2}uT = 2R$.

Hence locus of all points where shell lands on ground shifts along x-axis by 2R as shown in fig. (2).

From the fig (2).

$BC = \sqrt{R^2 + (2R)^2}$

$= \sqrt{5R^2} = \sqrt{5}R$

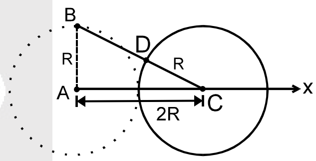


fig.2

Hence the minimum required distance is

$BD = BC - DC = \sqrt{5}R - R =$ **$(\sqrt{5}-1)$ R Ans.**

12. $v_0 = \sqrt{gH\left(1+\dfrac{\ell^2}{H^2}\right)}$, $\tan\alpha = \dfrac{H}{\ell}$

13. (a) 1s (b) $5\sqrt{3}$ m. 5 m

14. 9 **15.** 50

16. width of the roof is 9.6 m

$\theta = \tan^{-1} 3/2$ or $\theta = 45°$

17. $\dfrac{\sqrt{3620}}{3}$ m/s, $\tan^{-1}\left(\dfrac{9}{10}\right)$

SOLUTIONS OF PROJECTILE MOTION
EXERCISE-1
PART - I

SECTION (A)

A-1. $T_1 = \frac{2u\sin\theta}{g}$; $T_2 = \frac{2u\sin(90-\theta)}{g}$

$\frac{T_1}{T_2} = \frac{\sin\theta}{\sin(90-\theta)}$ = tan θ

{or $T_1 : T_2 = \tan\theta : 1$

A-2. $H = \frac{u^2\sin^2\theta}{2g}$, $H_1 = \frac{u^2\sin^2\theta}{2g}$, $H_2 = \frac{u^2\sin^2(90-\theta)}{2g}$

$\frac{H_1}{H_2} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$ $H_1 : H_2 = \tan^2\theta : 1$

A-3. Horizontal Range $R = \frac{u^2\sin2\theta}{g}$

Vertical height $H = \frac{u^2\sin^2\theta}{2g}$

given R = H

So $\frac{u^2\sin2\theta}{g} = \frac{u^2\sin^2\theta}{2g}$

$2\times2\sin\theta\cos\theta = \sin^2\theta$

$\tan\theta = 4$

A-4. R same for θ_1 & θ_2

$\theta_2 = 90 - \theta_1$

$T = \frac{2u\sin\theta}{g}$ ∴ $T_1 = \frac{2u\sin\theta}{g}$; $T_2 = \frac{2u\sin(90-\theta)}{g}$

&, $R = \frac{u^2\sin2\theta}{g}$; $T_1T_2 = \frac{2^2\times u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left(\frac{u^2\sin2\theta}{g}\right)$

$T_1T_2 = \frac{2R}{g}$ **Ans**

A-5. R_{max} = 100 m (given) H_{max} = ? (for any θ)

$R_{max} = \frac{u^2\sin90}{g} = 100 \Rightarrow u^2 = 1000$ (θ = 45º for maximum range)

∴ $(H)_{max} = \frac{u^2(\sin^2\theta)_{max}}{2g} = \frac{u^2}{2g}$ (θ = 90º for maximum height)

$= \frac{1000}{20}$

⇒ H_{max} = 50 m **Ans**

A-6. (1) $\theta = 45^\circ$ $u = 20$ m/s

$$T = \frac{2u_y}{g} = \frac{2 \times 20 \times \frac{1}{\sqrt{2}}}{10} = 2\sqrt{2}\text{ s} \qquad u_x = 20 \times \frac{1}{\sqrt{2}}$$

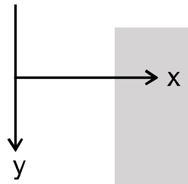
$$\text{Now, } R = \left(20 \times \frac{1}{\sqrt{2}}\right) \times 2\sqrt{2} = 40\text{ m}$$

$\Rightarrow$ The man should come (travel) 60 – 40 = 20 m

$$\text{time } 2\sqrt{2}\text{ s \& vel} = \frac{20\text{m}}{2\sqrt{2}\text{s}} = 5\sqrt{2}\text{ m/s}$$

SECTION (B) :

B-1. $u_x = 98$ m/s



(i) $H = 490$ m, $g = 9.8$ m/s^2, $u_y = 0$, $a_y = g = 9.8$ m/s^2

$s_y = u_y t + 1/2 a_y t^2$,

$\therefore$ $490 = 0 + 1/2 \times 9.8\, t^2$, $100 = t^2$ $\Rightarrow$ $t = \pm 10$

Ignoring "–ve" value, as it gives time before the time of projection, we get t = 10 s **Ans**

(ii) Distance from the hill = $u_x \times T = 98 \times 10 = 980$ m **Ans**

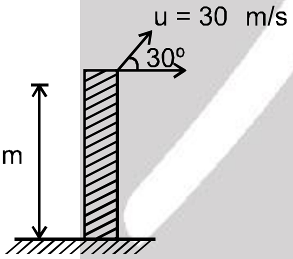
(iii) $V = \sqrt{V_x^2 + V_y^2}$ $V_x = u_x = 98$ m/s $V_y^2 = u_y^2 + 2a_y s_y$

$V_y^2 = 0 + 2 \times 9.8 \times 490$,

So $V = \sqrt{98^2 + 2 \times 9.8 \times 490}$,

$V = 98\sqrt{2}$ m/s. **Ans**

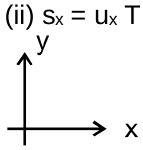
B-2.



$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{30 \times 30 \times \frac{1}{2} \times \frac{1}{2}}{2 \times 10} = \frac{90}{8} = 11.25$$

$\therefore$ H from ground H = 50 + 11.25 = 61.25 m. **Ans**

(ii) $s_x = u_x T + a_x T^2$, $a_x = 0$ $\Rightarrow$ $s_x = u_x T$



To find T $s_y = u_y T + \frac{1}{2} a_y T^2$ Where, $s_y = -50$ = vertical displacement

$T\ s_y = u_y T + \frac{1}{2} a_y T^2$ $s_y = -50 =$

$u_y = u \sin 30^\circ = 15$ m/s , $a_y = -g = -10$ m/s^2

Substituting these values,

$-50 = 15T + \frac{1}{2}(-10)T^2$; or $T^2 - 3T - 10 = 0$; or, $T^2 - 5T + 2T - 10 = 0$;

or, $T(T-5) + 2(T-5) = 0$; or $(T-5)(T+2) = 0$; or, $T = 5$ or $T = -2$

$\Rightarrow$ T = 5 sec **Ans**

$s_x = u\cos\theta \,.\, T = 30 \times \cos 30^\circ \times T = 30 \times \frac{\sqrt{3}}{2} \times 5 = 75\sqrt{3}$ m **Ans**

SECTION (C) :

C-1. $y = \sqrt{3}\,x - g\frac{x^2}{2}$, from the given (above) eq. with the standard equation of trajectory

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

we get $\sqrt{3} = \tan\theta \Rightarrow \theta = 60^\circ$

$u^2\cos^2\theta = 1$, Putting $\theta = 60^\circ$ we get $u^2 = \frac{1}{(1/2)^2} \Rightarrow u = 2$m/s.

Alternate Solution

$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$

In this eq. at t = 0, x = 0, y = 0 ; $a_x = 0$; $a_y = -g$

using these conditions in the given equation we get.

$$\frac{dy}{dx} = \sqrt{3} - \frac{1}{2}g2x\frac{dx}{dx}$$

To find θ, we now find $\tan\theta = \left.\frac{dy}{dx}\right]_{at\ t=0}$

$\tan\theta = \left.\frac{dy}{dx}\right]_{at\ t=0}$

$\therefore \left.\frac{dy}{dx}\right]_{t=0} = \sqrt{3} - 0$ $\{\because x = 0 \text{ at } t = 0\}\{\because t = 0\ x = 0\}$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$ **Ans.**

$$\frac{dy}{dt} = \sqrt{3}\frac{dx}{dt} - \frac{1}{2}g\left[2x\left(\frac{dx}{dt}\right)\right]$$

$V_y = \sqrt{3}\ V_x - gx\,V_x$

At t = 0, x = 0, $V_y = u_y$ & $V_x = u_x$; $u_y = \sqrt{3}\ u_x$

$\frac{d^2y}{dt^2} = \sqrt{3}\frac{d^2x}{dt^2} - g\left[x\frac{d^2x}{dt^2} + \frac{dx}{dt}\times\frac{dx}{dt}\right]$ here $a_x = \frac{d^2x}{dt^2} = 0$

$\Rightarrow \frac{d^2y}{dt^2} = \sqrt{3}\times 0 - g\ [0 + V_x]^2 \Rightarrow a_y = -g\,V_x^2$

Now, $a_y = -g \Rightarrow V_x^2 = 1 \Rightarrow V_x = \pm 1$

$v_x = u_x + a_x t$, $a_x = 0 \Rightarrow v_x = u_x$

$\therefore u_x = \pm 1 \Rightarrow u_y = \sqrt{3}(\pm 1)$; $u_y = \pm\sqrt{3}$

$\therefore$ Speed $= u = \sqrt{u_x^2 + u_y^2} = \sqrt{(\pm 1)^2 + (\pm\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4}$; u = 2 m/s. **Ans**

C-2 $y = x \tan\theta\,(1 - x/R)$

$\Rightarrow \quad \frac{R}{4} = \frac{3R}{4}\tan\theta\left(1 - \frac{3R}{4R}\right) \qquad \Rightarrow \quad 1 = 3\tan\theta\,(1/4)$

$\Rightarrow \quad \tan\theta = 3/4 \qquad \Rightarrow \quad \theta = 53°$

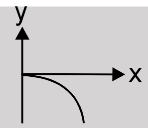
C-3. Comparing $\vec{r} = at\hat{i} - bt^2\hat{j}$ with $\vec{r} = x\hat{i} + y\hat{j}$

$\vec{r} = at\hat{i} - bt^2\hat{j} \quad \vec{r} = x\hat{i} + y\hat{j}$

we get $x = at$

& $y = -bt^2$

$\Rightarrow \quad y = -b\left(\frac{x}{a}\right)^2$ equation of trajectory



(i) $y = -\frac{bx^2}{a^2}$ **Ans**

(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$, acceleration $= -2b\hat{j}$,

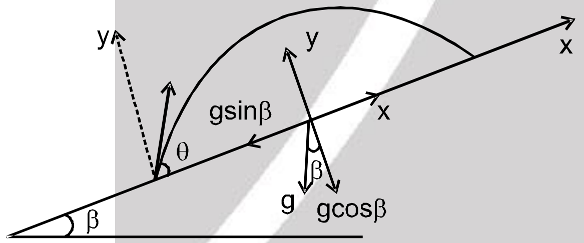
$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, |acceleration| $= 2b$

$\vec{v} = a\hat{i} - 2bt\hat{j}, = -2b\hat{j}$,

$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, $= 2b$

SECTION (D) :

D-1.



(a) a_x = x component of acceleration = $-g\sin\beta$

(b) y – component of acc^n = $a_y = -g\cos\beta$

(c) Let x – component of vel = V_x

(d) Let y – component of vel = V_y

(e) Let x-component of displacement = s_x

(f) Let s_y = y – component of displacement

PART - II

SECTION (A) :

A-1. $V = u + at$

V_y reduces then increases

$\Rightarrow$ V reduces then increase then increase ($\because$ V_x is constant V_x)

$\Rightarrow$ Speed first reduces then increases. So "A" is not correct

"A"

KE = $\frac{1}{2}mV^2 = \frac{m}{2}$ (speed)^2 ⇒ "B" is not correct " B"

V_y = changes ⇒ "C" is not correct. "C"

V_x = constt. since gravity is vertically down

V_x =

⇒ no component of acceleration along the horizontal direction.

⇒ "D" is correct. "D" **Ans**

A-2. In projectile motion Horizontal acceleration a_x = o & Vertical acceleration a_y = g =10m/s^2

a_x= o a_y = g =10m/s^2

$a_x = 0$

a_y = 10 (down)

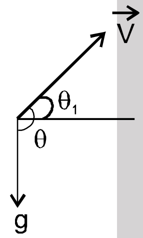
⇒ only "C" is correct "C" **Ans**

A-3. Acute Angle of Velocity with horizontal possible is – 90º to + 90º hence angle with g is 0° to 180º.

– 90º + 90º g 0° 180º

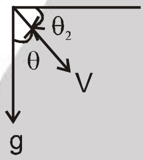
θ_1 is acute θ_1

⇒ $0^o \le \theta_1 < 90^o$ (during the upward journey of mass)



from fig $\theta = 90^o + \theta_1$

or, $90^o \le \theta < 180^o$ ....(1)



During downward motion

$0^o < \theta_2 < 90^o$

$\theta = 90^o - \theta_2$

$0^o < \theta < 90^o$ ....(2)

From eq. (1) and (2)

i.e., $0 < \theta < 90^o$ U $90^o \le \theta < 180^o$

⇒ $0^o < \theta < 180^o$

"D" **Ans.**

A-4. Avg. vel. b/w A & B = $\frac{\vec{v}_1 + \vec{v}_2}{2}$ (Acceleration is constant = g)

Now, if $\vec{v}_1 = V_1\hat{i} + V_1\hat{j}$

Than $\vec{v}_2 = V_1\hat{i} - V_1\hat{j}$ (both A & B are at same lavel)

∴ $\vec{v}_{avg.} = V_1\hat{i} = V\sin\theta$ (θ is from vertical) " B " **Ans.**

A-5. $y = ax^2$ .....(1)

given $V_x = c$

from (1) $\frac{dy}{dt} = 2a\,x.\,\frac{dx}{dt}$

$V_y = 2ax\,.c$ .....(2)

from (2) $\frac{dv_y}{dt} = 2ac.\,\frac{dx}{dt}$

$a_y = 2acV_x$

$a_y = 2ac^2$

$\vec{a}_y = 2ac^2\,\hat{j}$

A-6. Gravitational acceleration is constant near the surface of the earth.

A-7. At maximum height $v = u\cos\theta$

$$\frac{u}{2} = v \Rightarrow \frac{u}{2} = u\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{u^2\sin(120°)}{g} = \frac{u^2\cos 30°}{g} = \frac{\sqrt{3}u^2}{2g}$$

A-8. $\vec{u}_x = 6\,\hat{i} + 8\,\hat{j}$

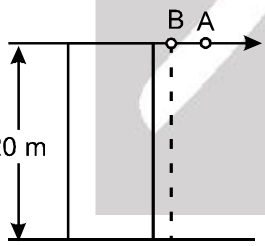
$\vec{u}_x = 6\hat{i}$

$u_y = 8\,\hat{j}$

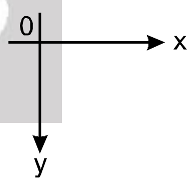
$$R = \frac{2u_xu_y}{g} = \frac{2\times6\times8}{10} = 9.6$$

SECTION (B) :

B-1.



$10\text{ m/s} = u_A$; $u_B = 0$ m/s $u_A = 10\,\hat{i}$



On reaching the ground,

Both will have same vertical velocity $V_y^2 = u_y^2 + 2a_y\,s_y$

since $u_y = 0$ for both A & B

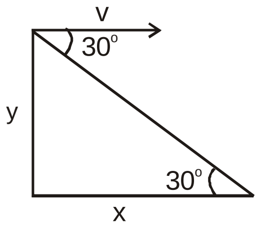
$a_y = g$ for both A & B

$s_y = 20$ m for both A & B

Thats why the time taken by both are same

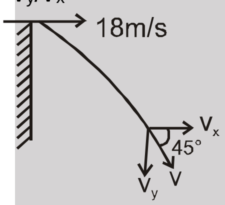
B-2. AC = $\frac{1}{2}gt^2 = 45$ m BC = $45\sqrt{3}$ m = u.t u = $\frac{45}{\sqrt{3}} = 15\sqrt{3}$ m/s.

Alter : Object is thrown horizontally so $u_x = v$ & $u_y = 0$
from Diagram`



$-y = u_y t - 1/2\ gt^2$; $y = 1/2 \times 10 \times (3)^2$
y = 45m ....(1)
& tan 30° = y/x= > y= $\sqrt{3}$ x ....(2)
& x = v t = 3v ....(3)
from equation (1), (2) & (3) `
$45\sqrt{3} = 3v$; $v = 15\sqrt{3}$ m/s

B-3. tan45° = v_y/v_x



⇒ $v_y = v_x = 18$m/s **Ans.**

B-4. In 2 sec. horizontal distance travelled by bomb = 20 × 2 = 40 m.
In 2 sec. vertical distance travelled by bomb = 1/2 × 10 × 2^2 = 20 m.
In 2 sec. horizontal distance travelled by Hunter = 10 × 2 = 20 m.

Time remaining for bomb to hit ground = $\sqrt{\frac{2 \times 80}{10}} - 2 = 2$ sec.

Let V_x and V_y be the velocity components of bullet along horizontal and vertical direction.

Thus we use, $\frac{2V_y}{g} = 2 \Rightarrow V_Y = 10$ m/s and $\frac{20}{V_x - 20} = 2 \Rightarrow V_x = 30$ m/s

Thus velocity of firing is $V = \sqrt{V_x^2 + V_y^2} = 10\sqrt{10}$ m/s.

2 sec. = 20 × 2 = 40 m.
2 sec. = 1/2 × 10 × 2^2 = 20 m.
2 sec. = 10 × 2 = 20 m.

$= \sqrt{\frac{2 \times 80}{10}} - 2 = 2$ sec.

V_x

$\frac{2V_y}{g} = 2 \Rightarrow V_Y = 10$ m/s $\frac{20}{V_x - 20} = 2 \Rightarrow V_x = 30$ m/s

$V = \sqrt{V_x^2 + V_y^2} = 10\sqrt{10}$ m/s.

SECTION (C) :

C-1. For (Y_{max}) $\Rightarrow$ $dY/dt = 0$

$$\Rightarrow \quad \frac{d}{dt}(10\,t - t^2) = 10 - 2\,t \quad \Rightarrow \quad t = 5$$

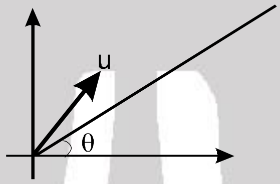
$$\Rightarrow \quad Y_{max} = 10(5) - 5^2 = 25 \text{ m}$$

Ans "D"

SECTION (D) :

D-1. On the incline plane the maximum possible Range is

$$R = \frac{V^2}{g(1+\sin\theta)}$$

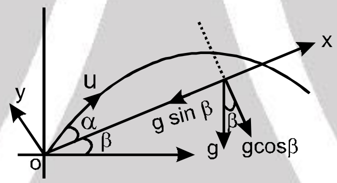


Range max = ?

Let $\theta = \beta$

And angle of projection from the inclined plane = α

= α



$u_y = u \sin\alpha$ $\quad u_x = \cos\alpha$ $\quad a_x = -g\sin\beta$ $\quad a_y = -g\cos\beta$

Range = $s_x = u_xT + \frac{1}{2}a_xT^2$

(on the inclined plane) where $T = \frac{2u_y}{g_y}$

$$\Rightarrow \quad s_x = (u\cos\alpha)\left[\frac{2u\sin\alpha}{g\cos\beta}\right] + \frac{1}{2}[-g\sin\beta]\left[\frac{2u\sin\alpha}{g\cos\beta}\right]^2$$

$$= \frac{2u^2\sin\alpha}{g\cos^2\beta}[\cos\alpha\cos\beta - \sin\alpha\sin\beta]$$

$$s_x = \frac{2u^2\sin\alpha}{g\cos^2\beta}[\cos(\alpha+\beta)] = \frac{u^2}{g\cos^2\beta}[2\sin\alpha\cos(\alpha+\beta)] = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha+\beta)+\sin(-\beta)]$$

$$s_x = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha+\beta)-\sin\beta]$$

Now s_x is max s_x

when $\sin(2\alpha+\beta)$ is max $\quad (\because \beta = \text{constt.})$

$\sin(2\alpha+\beta)$ $\quad (\because \beta =$ अचर$)$

$$\Rightarrow \quad 2\alpha + \beta = \pi/2 \quad \Rightarrow \quad \alpha = \frac{\left(\frac{\pi}{2} - \beta\right)}{2}$$

i.e., when ball is projected at the angle bisector of angle formed by inclined plane and dir. of net accelaration reversed.

& $(s_x)_{max} = \dfrac{u^2}{g\cos^2\beta}\left[1-\sin\beta\right] = \dfrac{u^2 \quad (1-\sin\beta)}{g \quad (1-\sin\beta) \quad (1+\sin\beta)}$

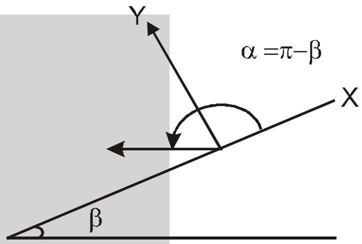
Max. Range on an inclined plane $= \dfrac{u^2}{g(1+\sin\beta)}$

Here $\beta = \theta$

$\Rightarrow \qquad R_{max} = \dfrac{u^2}{g(1+\sin\theta)}$ **Ans** "B"

D-2.

Sol. $R = \dfrac{v^2}{g\cos^2\beta}\left[\sin(2\alpha+\beta)-\sin\beta\right]$



Putting $\beta = 45^o$ & $\alpha = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

$R = \dfrac{v^2}{g\left(\dfrac{1}{\sqrt{2}}\right)^2}\left[\sin\left(2\times\dfrac{3\pi}{4}+\dfrac{\pi}{4}\right)-\sin\dfrac{\pi}{4}\right]$

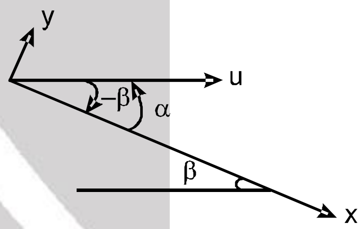
$s_x = \dfrac{v^2\times 2}{g}\left[-2\times\dfrac{1}{\sqrt{2}}\right] = -2\sqrt{2}\dfrac{v^2}{g}$

(–ve sign indicates that the displacement is in –ve x direction)

$\Rightarrow \qquad \text{Range} = 2\sqrt{2}\dfrac{v^2}{g}$ **Ans** "D"

Alternate II method

$\beta = -\dfrac{\pi}{4}$ & $\alpha = \dfrac{\pi}{4}$



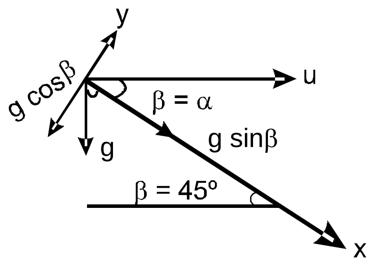
$R = \dfrac{u^2}{g \ \cos^2\beta}\left[\sin \ (2\alpha+\beta)-\sin\beta\right]$

$= \dfrac{u^2}{g\left(\dfrac{1}{\sqrt{2}}\right)^2}\left[\sin\dfrac{\pi}{4}-\sin\left(-\dfrac{\pi}{4}\right)\right]$

$R = \dfrac{2\sqrt{2}u^2}{g}$ (along +ve x die.) (+ve $\hat{x}$)

III Method

$u_x = u\cos\beta$, $\quad T = \left|\dfrac{2u\sin\beta}{g\cos\beta}\right|$, $a_x = g\sin\beta$; $a_y = -g\cos\beta$



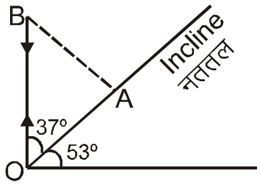
$s_x = u_x t + \dfrac{1}{2} a_x t^2 = (u\cos\beta)\left[\dfrac{2u\sin\beta}{g\cos\beta}\right] + \dfrac{1}{2}(g\sin\beta)\left(\dfrac{2u\sin\beta}{g\cos\beta}\right)^2$

Let $\alpha = \beta = 45^o$

So, $s_x = \dfrac{u^2 2}{g}\left(\dfrac{1}{\sqrt{2}}\right) + \dfrac{1}{2}g\left(\dfrac{1}{\sqrt{2}}\right)\dfrac{2.2u^2}{g^2} = \dfrac{2}{\sqrt{2}}\dfrac{u^2}{g}[1+1]$ $s_x = 2\sqrt{2}\dfrac{u^2}{g}$

Ans "D"

D-3. OB = $\dfrac{u^2}{2g}$ = 5m

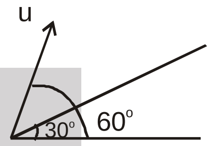


$\therefore$ AB = OB sin37º = 3m.

D-4.

u = 10m/s

Time of flight on the incline plane



T= $\dfrac{2u\sin\alpha}{g\cos\beta}$

given α =30º & β = 30º & u = $10\sqrt{3}$ m/s

T = $\dfrac{2\times10\sqrt{3}\sin30^{o}}{10\cos30^{o}}$

So, T= 2 sec .

D-5. H = $\dfrac{u_{\perp}^2}{2a_{\perp}}$

$a_{\perp}$ is same for all the three cases.

$H_A = \dfrac{(u\sin\alpha)^2}{2a_{\perp}}$, $H_B = \dfrac{u^2}{2a_{\perp}}$ and $H_C = \dfrac{(u\cos\alpha)^2}{2a_{\perp}}$ $\therefore$ $H_B = H_A + H_C$

PART - III

1. Time of flight T = $\dfrac{2u}{g\cos45^\circ} = \dfrac{2u}{g\cos45^\circ} = \dfrac{2\sqrt{2}u}{g}$

$\therefore$ D $\rightarrow$ p

Velocity of stone is parallel to x-axis at half the time of flight.

$\therefore$ A $\rightarrow$ r

At the instant stone make 45° angle with x-axis its velocity is horizontal.

$\therefore$ The time is = $\dfrac{u\sin45^\circ}{g} = \dfrac{u}{\sqrt{2}g}$

$\therefore$ B $\rightarrow$ s

The time till its displacement along x-axis is half the range is = $\dfrac{1}{\sqrt{2}}T = \dfrac{2u}{g}$

$\therefore$ C $\rightarrow$ q

T = $\dfrac{2u}{g\cos45^\circ} = \dfrac{2u}{g\cos45^\circ} = \dfrac{2\sqrt{2}u}{g}$ $\therefore$ D $\rightarrow$ p

$\therefore$ A $\rightarrow$ r

= $\dfrac{u\sin45^\circ}{g} = \dfrac{u}{\sqrt{2}g}$ $\therefore$ B $\rightarrow$ s

= $\dfrac{1}{\sqrt{2}}T = \dfrac{2u}{g}$ $\therefore$ C $\rightarrow$ q

2. Equation of path is given as $y = ax - bx^2$

Comparing it with standard equation of projectile;

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \qquad \tan\theta = a,\ \frac{g}{2u^2\cos^2\theta} = b$$

Horizontal component of velocity = $u\cos\theta = \sqrt{\frac{g}{2b}}$

Time of flight $T = \frac{2u\sin\theta}{g} = \frac{2(u\cos\theta)\tan\theta}{g} = \frac{2\left(\sqrt{\frac{g}{2b}}\right)a}{g} = \sqrt{\frac{2a^2}{bg}}$

Maximum height $H = \frac{u^2\sin^2\theta}{2g} = \frac{[u\cos\theta.\tan\theta]^2}{2g} = \frac{\left[\sqrt{\frac{g}{2b}}.a\right]^2}{2g} = \frac{a^2}{4b}$

Horizontal range $R = \frac{u^2\sin 2\theta}{g} = \frac{2(u\sin\theta)(u\cos\theta)}{g} = \frac{2\left[\sqrt{\frac{g}{2b}}.a\right]\left[\sqrt{\frac{g}{2b}}\right]}{g} = \frac{a}{b}$

$y = ax - bx^2$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\tan\theta = a,\ \frac{g}{2u^2\cos^2\theta} = b$$

$= u\cos\theta = \sqrt{\frac{g}{2b}}$

$T = \frac{2u\sin\theta}{g} = \frac{2(u\cos\theta)\tan\theta}{g} = \frac{2\left(\sqrt{\frac{g}{2b}}\right)a}{g} = \sqrt{\frac{2a^2}{bg}}$

$H = \frac{u^2\sin^2\theta}{2g} = \frac{[u\cos\theta.\tan\theta]^2}{2g} = \frac{\left[\sqrt{\frac{g}{2b}}.a\right]^2}{2g} = \frac{a^2}{4b}$

$R = \frac{u^2\sin 2\theta}{g} = \frac{2(u\sin\theta)(u\cos\theta)}{g} = \frac{2\left[\sqrt{\frac{g}{2b}}.a\right]\left[\sqrt{\frac{g}{2b}}\right]}{g} = \frac{a}{b}$

EXERCISE-2
PART - I

1. $a_x = 2$ m/s^2 ; $a_y = 0$

$u_x = 8$ m/s

$u_y = -15$ m/s.

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

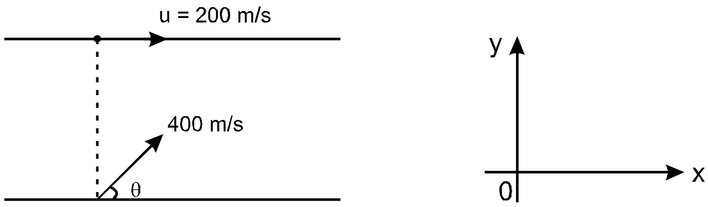
$V_y = u_y + a_y t$

$\Rightarrow$ $V_y = -15$ m/s

$V_x = u_x + a_x t$

$V_x = 8 + 2t$ $\Rightarrow$ $V = [(8 + 2t)\hat{i} - 15\hat{j}]$ m/s. **Ans.**

2.



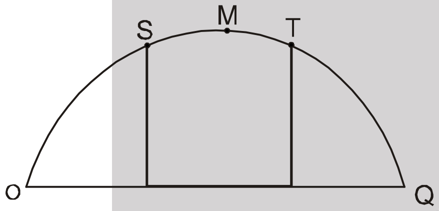
To hit, $400 \cos \theta = 200$

{$\because$ Both travel equal distances along horizontal, from their start and coordinates of x axis are same}

$\Rightarrow$ $\theta = 60^o$ **Ans.**

3. $$\frac{R^2}{8h} + 2h = \frac{\left(\frac{u^2 2\sin\theta\cos\theta}{g}\right)^2}{\frac{8 \times u^2 \sin^2\theta}{2g}} + 2\,\frac{u^2\sin^2\theta}{2g} = \frac{u^2}{g} \text{(max. horizontal Range)}$$

4.



$t_{(OS)} = 1$ sec

$t_{(OT)} = 3$

or $t_{(ST)} = 1/2\ t_{(OT)} - t_{(OS)} = 3 - 1 = 2$ sec

$\therefore$ $t_{(SM)} = t_{(ST)} = 1$ sec.

$\therefore$ $t_{(OM)} = t_{(OS)} + t_{(SM)} = 1 + 1 = 2$sec.

$\therefore$ Time of flight = 2 × 2 = 4 sec. **Ans. "C"**

5. Let initial and final speeds of stone be u and v.

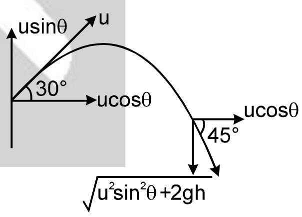
$\therefore$ $v^2 = u^2 - 2gh$ ....(1)

and $v \cos 30° = u \cos 60°$ ....(2)

solving 1 and 2 we get

$u = \sqrt{3gh}$

6.



Using $v = \sqrt{u^2 + 2gh}$

$v = \sqrt{u^2 \sin^2\theta + 2gh}$ (vertical comp. when striking)

Now tan 45° = 1

$u \cos\theta = \sqrt{u^2 \sin^2\theta + 2gh}$

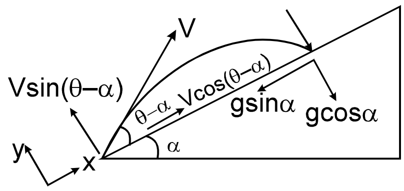
$u^2 \cos^2\theta = u^2 \sin^2\theta + 2gh$ ......(1)

$u^2 \left(\frac{3}{4} - \frac{1}{4}\right) = 2gh$

$u^2 = 4gh$

$u = 2\sqrt{gh}$; $\tan\theta = \frac{v_T}{v_H} = \frac{\sqrt{4gh.\frac{3}{4} + 2gh}}{2\sqrt{gh} \times \frac{1}{2}} = \frac{\sqrt{5gh}}{\sqrt{gh}} = \sqrt{5}$

7.



Applying equation of motion perpendicular to the incline for y = 0.

$0 = V \sin(\theta - \alpha)t + \frac{1}{2}(-g\cos\alpha)\, t^2$

$\Rightarrow \quad t = 0 \quad \& \quad \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$

At the moment of striking the plane, as velocity is perpendicular to the inclined plane hence component of velocity along incline must be zero.

$0 = v\cos(\theta - \alpha) + (-g\sin\alpha).\ \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$

$v\cos(\theta - \alpha) = \tan\alpha.\ 2V\sin(\theta - \alpha)$

$\cot(\theta - \alpha) = 2\tan\alpha$ **Ans. (D)**

8. Since time of flight depends only on vertical component of velocity and acceleration. Hence time of flight is

$T = \frac{2u_y}{g}$ where $u_x = u\cos\theta$ and $u_y = u\sin\theta$

$\therefore$ In horizontal (x) direction

$d = u_x t + \frac{1}{2} a_x t^2 = u\cos\theta\left(\frac{2u\sin\theta}{g}\right) + \frac{1}{2}g\left(\frac{2u\sin\theta}{g}\right)^2$

$= \frac{2u^2}{g}(\sin\theta\cos\theta + \sin^2\theta)$

We want to maximise f(θ)

$f(\theta) = \cos\theta\sin\theta + \sin^2\theta$

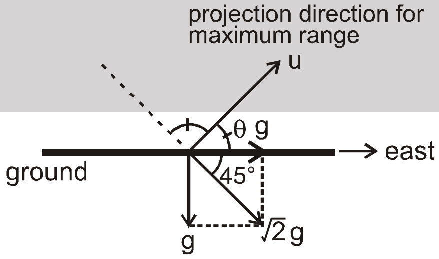
$\Rightarrow \quad f'\theta = -\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 0$

$\Rightarrow \quad \cos 2\theta + \sin 2\theta = 0 \quad \Rightarrow \quad \tan 2\theta = -1$

or $\quad 2\theta = \frac{3\pi}{4} \quad$ or $\quad \theta = \frac{3\pi}{8} = 67.5°$

Alternate :

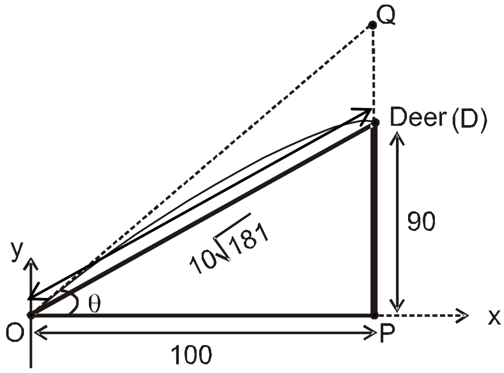
As shown in figure, the net acceleration of projectile makes on angle 45° with horizontal. For maximum range on horizontal plane, the angle of projection should be along angle bisector of horizontal and opposite direction of net acceleration of projectile.



$\therefore \quad \theta = \frac{135°}{2} = 67.5°$

PART - II

1. OD = $10\sqrt{181}$ PD = 90



Let θ be angle of projection

we have QD = PQ – PD.

Also from equation of trajectory

$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2}\sec^2\theta$

Coordinate of deer ≡ (100,90)

$\therefore \quad 90 = 100\tan\theta - \frac{5(100)^2}{(100)^2}\sec^2\theta$

or $\quad 90 = 100\tan\theta - 5(1 + \tan^2\theta)$

or $\quad \tan^2\theta - 20\tan\theta + 19 = 0$

or $\quad \tan^2\theta - 19\tan\theta - \tan\theta + 19 = 0$

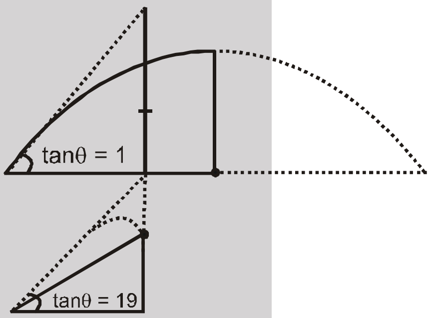
or $\quad \tan\theta(\tan\theta - 19) - 1(\tan\theta - 1) = 0$

or $\quad \tan\theta = 1$

$\quad \tan\theta = 19$

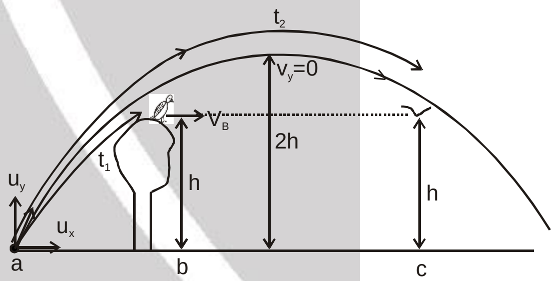
or $\quad \frac{PQ}{100} = 1$ for $\tan\theta = 1$

$\quad \frac{PQ}{100} = 19$ for $\tan\theta = 19$



The two roots signify the two possible trajectories for which the hit would be successful.

2. ac = ab + bc



bc = ac – ab

$V_Bt_2 = U_xt_2 - U_xt_1$

(V_B = velocity of bird)

$V_Bt_2 = U_x(t_2 - t_1)$ .... (1)

displacement in y direction in time 't'

$S_y = U_yt - 1/2\ gt^2$

$\therefore\ V_y^2 = U_y^2 - 2g(2h)$

$0 = U_y^2 - 2g(2h)$

$U_y = \sqrt{4gh}$

$h = \sqrt{2g(2h)}\,t - \frac{1}{2}gt^2$

on solving we get

$t_1 = \sqrt{h/g}(2-\sqrt{2})$

$t_2 = \sqrt{h/g}(2+\sqrt{2})$

On putting the value of t_1 and t_2 in equation (1)

we get $\frac{U_x}{V_B} = \frac{\sqrt{2}+1}{2}$

Aliter

Time for stone to move from b to c = $2\sqrt{\frac{h}{g}}$

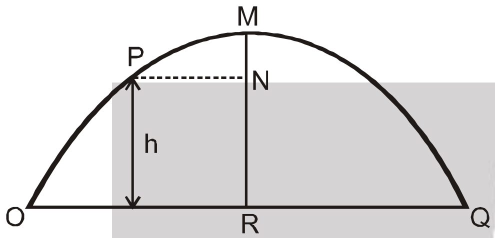
Time for bird to fly from b to c = $\sqrt{2\frac{h}{g}} + \sqrt{\frac{h}{g}}$

Therefore equating the distance bc from both the cases

$$V_B\left(\sqrt{\frac{2h}{g}} + \sqrt{\frac{h}{g}}\right) = U_x\left(2\sqrt{\frac{h}{g}}\right)$$

$$\frac{U_x}{V_B} = \frac{(\sqrt{2}+1)}{2}$$ **Ans.**

3.



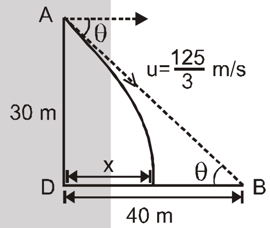
$t_{op} = 4$, $t_{PQ} = 5$ $\therefore$ $t_{OQ} = 4 + 5 = 9$

or $\tan\theta = 9/2 = 4.5$

$RM - MN = h = 1/2\ g\ [(4.5)^2 - (.5)^2]$

$= 1/2 \times 9.8 \times 20 = 98$

4. In ΔABD, $\tan\theta = \frac{30}{40} = \frac{3}{4}$



Let time taken be 't' in x-direction

$x = u_x t$

$x = u\cos\theta\ t$

$x = \frac{125}{3} \times \frac{4}{5} t \Rightarrow x = \frac{100}{3}\ t$ ....(1)

In y-direction

$y = u_y t + \frac{1}{2} gt^2$

$30 = u\sin\theta\ t + \frac{1}{2} gt^2$

$30 = \frac{125}{3} \times \frac{3}{5} t + 5t^2$

$t^2 + 5t - 6 = 0$

$(t + 6)(t - 1) = 0$

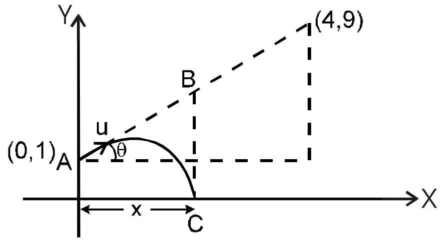
$t = 1$ sec. ....(2)

From (1) and (2)

$x = 100/3$

$\therefore$ Packet is short by a distance of $40 - \frac{100}{3} = \frac{20}{3}$ m **Ans.**

5. $\tan\theta = \frac{9-1}{4-0} = 2,$

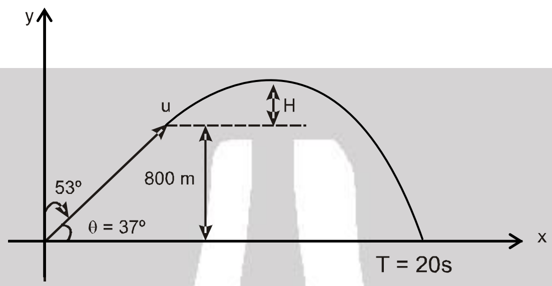


$y = u_y t + \frac{1}{2} a_y t^2$

now $-1 = u\sin\theta\ (1) - \frac{1}{2} g\ (1)^2$

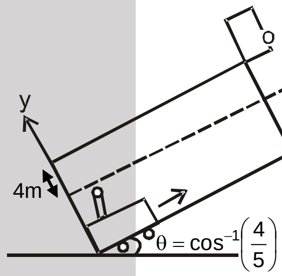
$u\sin\theta = 4$ and $\sin\theta = \frac{2}{\sqrt{5}}$ $\Rightarrow$ $u = 2\sqrt{5}$

6.



$s_y = u_y t + \frac{1}{2} a_y t^2$ $800 = (-u\cos 53^o)\ T + \left(\frac{10}{2}\right) T^2 \Rightarrow$ $u = 100$ m/s

7. At t = 0



a_y be the vertical component of acc^n of the ball w.r.t ground.

$a_y = -g\cos\theta = -g \times \frac{4}{5}$

while crossing through loop the velocity is parallel to x-axis

$\therefore$ $V_y = 0$

y co-ordinate of loop = + 4

$V_y^2 - u_y^2 = 2a_y\ (y_f - y_i)$

$0 - u_y^2 = -2.\ \frac{4g}{5}\ (4-0)$

$u_y^2 = \frac{8g}{5}\ .\ 4$

$u_y^2 = 8 \times 8$

$u_y = 8$ m/s

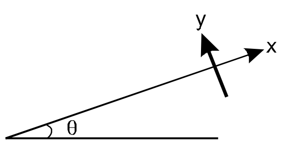
Time taken by the ball to reach the loop.

$V_y - u_y = a_y t$

$0 - 8 = -\frac{4g}{5}\ t$

or t = 1 second

II method : $V_y = 0$ (given)

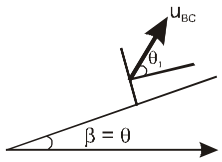


$V_y = u_y + a_y t$ ....(1)

$s_y = u_y t + \frac{1}{2} a_y t^2$ ...(2)

Two eq. two variable 'u_y' & 't'

to find 't' ; as shown below.



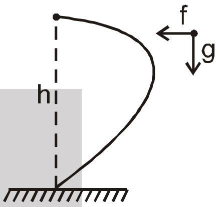
$V_y = u_y + a_y t$

$0 = u_{BC} \sin \theta_1 - g \cos \beta$ T. $\quad (\beta = \cos^{-1} 4/5 = 37^o)$

u_{BC} = vel. of ball wrt car. $\quad \Rightarrow \quad u_{BC} \sin \theta_1 = u_y = T. \times 10 \times 4/5 = 8T.$

$s_y = (8T)\, T + T^2 = 4, (s_y = 4\text{ m}) \quad \Rightarrow \quad T = 1s$ **Ans.**

8. Time taken to reach the ground is given by $h = \frac{1}{2} gt^2$ ....(1)



Since horizontal displacement in time t is zero

$\therefore \quad t = 2v/f$ ....(2)

$$h = \frac{2gv^2}{f^2}$$

9. At t = 0 $u_x = u\cos\theta$ and $u_y = u\sin\theta$

$\vec{u} = u\cos\theta\, \hat{i} + u\sin\theta\, \hat{j}$

Let after time 't' the velocity of projectile be v if its intial velocity is u

At time t

$V_x = u\cos\theta, \quad V_y = u\sin\theta - gt$

$\vec{V} = u\cos\theta\, \hat{i} + (u\sin\theta - gt)\, \hat{j}$

$u \perp v$

$\vec{u} \cdot \vec{v} = 0$

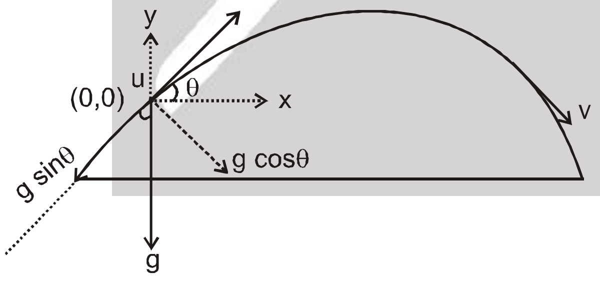
$(u\cos\theta\, \hat{i} + (u\sin\theta - gt)\, \hat{j})\, (u\cos\theta\, \hat{i} + u\sin\theta\, \hat{j})$

$u^2\cos^2\theta + (u\sin\theta)^2 - gt\, u\sin\theta = 0$

$u^2(\cos^2\theta + \sin^2\theta) = gtu\sin\theta$

$\frac{u}{g\sin\theta} = t$

ALTERNATE :



Now Let $\vec{u}$ be $\perp$ v after time t, then component of velocity along u becomes zero.

component of $\vec{g}$ along $\vec{u} = -g\sin\theta$

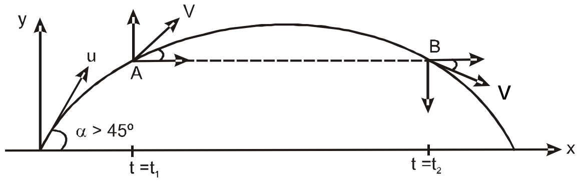
$= -g\sin\theta$

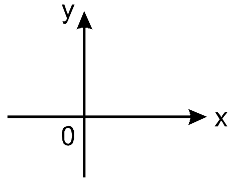
$\therefore \quad 0 = u - g\sin\theta\, t$

$t = u/g\sin\theta$

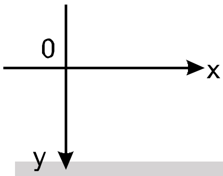
PART - III

1.





$V_x = V_y$ Now, we could have choosen coordinate axis as



⇒ Time at which $V_x = V_y$ is what we are solving

Now, $V_x = u \cos \alpha$

$V_y = u \sin\alpha - gt$

⇒ $u \cos \alpha = u \sin \alpha - gt$ $\{ \because V_y = V_x \}$; at $t = t_1$ (say)

⇒ $t_1 = \frac{u}{g} (\sin \alpha - \cos\alpha)$ **"C" Ans**

Also when $V_y \equiv V_x$ {i.e., when we choose 'y' axis as – y} at $t = t_2$ (say)

$u \cos \alpha = -(u \sin \alpha - gt_2)$

⇒ $t_2 = \frac{u}{g} (\sin \alpha + \cos \alpha)$ **"B" Ans**

2. $x = 24 = u \cos\theta.t$

⇒ $t = \frac{24}{24\cos\theta} = \frac{1}{\cos\theta}$

$y = 14 = u \sin\theta t - \frac{1}{2} gt^2$

⇒ $14 = \frac{u\sin\theta}{\cos\theta} - \frac{5}{\cos^2\theta}$ ⇒ $14 = u \tan\theta - 5 \sec^2\theta$

⇒ $5 \tan^2\theta - 24 \tan\theta + 19 = 0$ ⇒ $\tan\theta = 1, 19/5$. **Ans**

3. Since maximum heights are same, their time of flight should be same

∴ $T_1 = T_2$

Also, vertical components of initial velocity are same.

∴ Since range of 2 is greater than range of 1.

∴ Horizontal component of velocity of 2 > horizontal component of velocity of 1.

Hence, $u_2 > u_1$.

4. $R = \frac{u^2 \sin 2\theta}{g}$

$\sqrt{3} = 20.\frac{2\sin\theta\cos\theta}{10}$

$\sin\theta\sqrt{1-\sin^2\theta} = \frac{\sqrt{3}}{4}$ ⇒ $16\sin^4\theta - 16\sin^2\theta + 3 = 0$

$\sin^2\theta = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{3}{16}} \quad \Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}\ ;\ \frac{1}{2}$

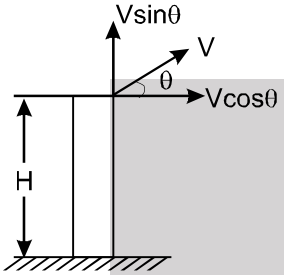
$\theta = 60^\circ\ ;\ 30^\circ$

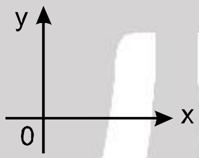
$H_{max} = \frac{u^2 \sin^2\theta}{2g} = 0.75\text{m} \ \&\ 0.25 \text{ m}$

$V_{min} = \sqrt{5}$ m/s, $\sqrt{15}$ m/s

$T = \frac{2u\sin\theta}{g} = \sqrt{\frac{3}{5}};\sqrt{\frac{1}{5}}$

5.





Let final vel be V_2

Now v_{2x} = horizontal component of velocity

$V_{2x} = V\cos\theta \quad \& \ V^2_{2y} = (V\sin\theta)^2 + 2(-g)(-H)$

$\therefore \quad V^2_{2y} = V^2\sin^2\theta + 2gH$

$\Rightarrow \quad V^2_2 = V^2_{2x} + V^2_{2y}$

$= (V\cos\theta)^2 + [V^2\sin^2\theta + 2gH]$

$V^2_2 = V^2 + 2gH$

i.e., $V_2 = \sqrt{V^2 + 2gH}$

This magnitude of final velocity is independent of θ

$\Rightarrow$ all particles strike the ground with the same speed.

i.e., 'A' is correct.

In vertical motion

The highest velocity (initial) along the direction of displacement is possessed by particle (1). Hence particle (1) will reach the ground earliest. [Since a_y and s_y are same for all]

i.e., 'C' is correct **Ans** A & C

PART - IV

1. Velocity at P is completely horizontal i.e. $u\cos\theta = 20\cos30^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$ m/sec.

$\therefore v_{vertical} = 0$ m/sec.

2. Assuming vertically downwards to be positive.

making equation along vertical direction (point A taken as reference)

$s = ut + \frac{1}{2}at^2$

$\therefore \quad 20 = -20\sin\theta \times t + 1/2 \times 10 \times t^2$

$\therefore \quad 20 = -20\sin30^\circ t + 5t^2$

$20 = -10t + 5t^2$

$\therefore \quad 5t^2 - 10t - 20 = 0 \quad$ or $\quad t^2 - 2t - 4 = 0$

$\therefore \quad t = \frac{2 \pm \sqrt{4+16}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$.

$\because$ at $(1 - \sqrt{5})$ sec the particle was at initial point on ground.

$\therefore$ accepted time = $(1 + \sqrt{5})$ sec

3. At point Q, x-component of velocity is zero. Hence, substituting in

$v_x = u_x + a_x t$

$0 = 10\sqrt{3} - 5\sqrt{3}t$

or $t = \dfrac{10\sqrt{3}}{5\sqrt{3}} = 2s$ **Ans.**

4. At point Q, $v = v_y = u_y + a_y t$

$\therefore$ $v = 0 - (5)(2) = -10$ m/s **Ans.**

Here, negative sign implies that velocity of particle at Q is along negative y direction.

5. PO = |displacement of particle along y-direction |

Here , $s_y = u_y t + \dfrac{1}{2} a_y t^2$

$= 0 - \dfrac{1}{2}(5)(2)^2 = -10$ m

$\therefore$ PO = 10 m

Therefore, $h = PO \sin 30° = (10)\left(\dfrac{1}{2}\right)$ or h = 5 m Ans.

6. Distance OQ = displacement of particle along x-direction = s_x

Here, $s_x = u_x t + \dfrac{1}{2} a_x t^2$

$= (10\sqrt{3})(2) - \dfrac{1}{2}(5\sqrt{3})(2)^2 = 10\sqrt{3}$ m

or $OQ = 10\sqrt{3}$ m

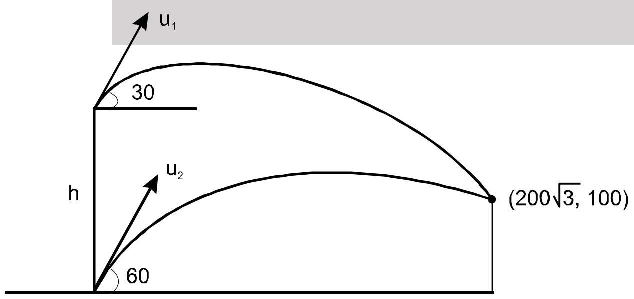
$\therefore$ $PQ = \sqrt{(PO)^2 + (OQ)^2} = \sqrt{(10)^2 + (10\sqrt{3})^2}$

$= \sqrt{100 + 300} = \sqrt{400}$

$\therefore$ PQ = 20 m **Ans.**

EXERCISE-3
PART - I

1.



$u_1 \cos 30^o = u_2 \cos 60^o$ (strike simultaneously)

$\sqrt{3}\, u_1 = u_2$

$100 = 200\sqrt{3} \tan 60^o - \dfrac{1}{2} \times \dfrac{g(200\sqrt{3})^2}{u_2^2 \cos^2 60^o}$ $\Rightarrow$ $u_2 = 40\sqrt{3}$ m/s from eq (1) and (2)

$u_1 = \frac{u_2}{\sqrt{3}}$ $\quad\quad$ $u_1 = 40$ m/s

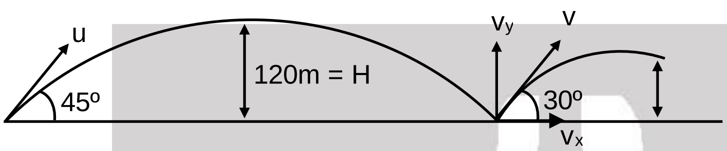
$x = u_2 \cos 60^o \times T$ $\quad$ $200\sqrt{3} = 40\sqrt{3} \times \frac{1}{2} \times T$ $\quad \Rightarrow \quad$ T = 10 sec

$\Rightarrow \quad (h - 100) = 200\sqrt{3}\ \tan 30^o - \frac{1}{2} g \frac{(200\sqrt{3})^2}{u_1^2 \cos^2 30^o}$

Putting g = 10 m/sec^2

& $\quad u_1 = 40$ m/sec $\quad\quad$ h = 400 m

2.



$K_2 = \frac{1}{2} mu^2$ $\quad\quad$ $\frac{v_y}{v_x} = \tan 30^o = \frac{1}{\sqrt{3}}$

$H = \frac{u^2 \sin^2 45^o}{2g}$ $\quad\quad$ $\sqrt{3}\, v_y = v_x$.

$= \frac{u^2}{4g} = 120$m $\quad\quad$ $K_f = \frac{1}{2} mv^2 = \frac{1}{2} m(v_x^2 + v_y^2)$

$K_f = \frac{1}{2} K_i$

$\Rightarrow \quad \frac{1}{2} m(v_x^2 + v_y^2) = \frac{1}{2} \times \frac{1}{2} mu^2$

$\Rightarrow \quad v_x^2 + v_y^2 = \frac{u^2}{2}$ $\quad\quad \Rightarrow \quad$ using $u_x = \sqrt{3}\, u_y$

$\Rightarrow \quad 3\ u_y^2 + u_y^2 = \frac{u^2}{2}$ $\quad\quad \Rightarrow \quad$ $u_y^2 = \frac{u^2}{8}$

$h = \frac{u_y^2}{2g} = \frac{u^2}{16g} = \frac{1}{4}\left(\frac{u^2}{4g}\right) = \frac{H}{4} = \frac{120}{4} = 30$ m

3. For first projectile

$< V > = \frac{R}{T} = U_x = v_1$

For journey

$$< V >_{1-n} = \frac{R_1 + R_2 + ... + R_n}{T_1 + T_2 + ... + T_n} = \frac{\frac{2u_{x_1}u_{y_1}}{g} + \frac{2u_{x_2}u_{y_2}}{g} + ... + \frac{2u_{x_n}u_{y_n}}{g}}{\frac{2u_{y_1}}{g} + \frac{2u_{y_2}}{g} + ... \frac{2u_{y_n}}{g}}$$

$$U_x\left[\frac{1+\frac{1}{\alpha^2}+\frac{1}{\alpha^4}+.....\frac{1}{\alpha^{2n}}}{1+\frac{1}{\alpha}+\frac{1}{\alpha^2}+....+\frac{1}{\alpha^n}}\right] = 0.8\ v_1 \Rightarrow \frac{v_0\left[\frac{1}{1-\frac{1}{\alpha^2}}\right]}{\left[\frac{1}{1-\frac{1}{\alpha}}\right]} = 0.8v_1$$

$\Rightarrow \quad \frac{\alpha}{1+\alpha} = 0.8 \quad \Rightarrow \quad \alpha = 4$

PART - II

1. $\vec{v} = \vec{u} + \vec{a}\ t$

$= (3\hat{i}+4\hat{j}) + (0.4\hat{i}+0.3\hat{j}) \times 10 = 7\ i + 7\ j |\ \vec{v}\ | = 7\sqrt{2}$

2. $\frac{d\vec{r}}{dt} = K(y\hat{i}+x\hat{j})$

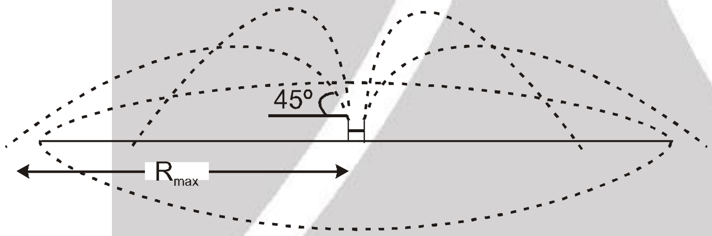
$\Rightarrow \quad dx/dt = y, \quad dy/dt = x$

So, $dy/dx = x/y$

$\int y\,dy = \int x\,dx \quad ; \quad \frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 = x^2 + \text{constant}$

3.



$R_{max} = \frac{v^2}{g}\ \sin 2\theta = \frac{v^2}{g}$

area $= \pi R^2$

$= \pi\frac{v^4}{g^2}$ **Ans.**

4. $h_{max} = \frac{u^2}{2g} = 10$

$u^2 = 200 \quad ....(1)$

$R_{max} = \frac{u^2}{g} = 20m$

5. $\vec{v} = \hat{i} + 2\hat{j}$

$\Rightarrow \quad x = t \quad ...(i)$

$y = 2t - 1/2\ (10t^2) \quad ...(ii)$

From (i) and (ii)

$y = 2x - 5x^2$

6. On inclined plane (range) R = $\frac{2u^2 \sin\alpha \cos(\alpha+\beta)}{g\cos^2\beta}$

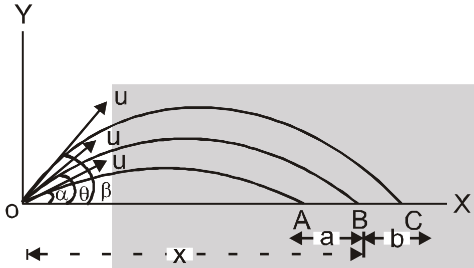
Where α = 15º, β = 30º, u = 2 m/s

On solving we get

R = $\frac{4}{5}\left(\frac{1}{\sqrt{3}} - \frac{1}{3}\right) \approx$ 20 cm

HIGH LEVEL PROBLEMS (HLP)

1.



OA = x – a = $\frac{u^2 \sin 2\alpha}{g}$ ...(1)

OC = x + b = $\frac{u^2 \sin 2\beta}{g}$ ...(2)

OB = x = $\frac{u^2 \sin 2\theta}{g}$ ...(3)

From eqs. (1) and (2)

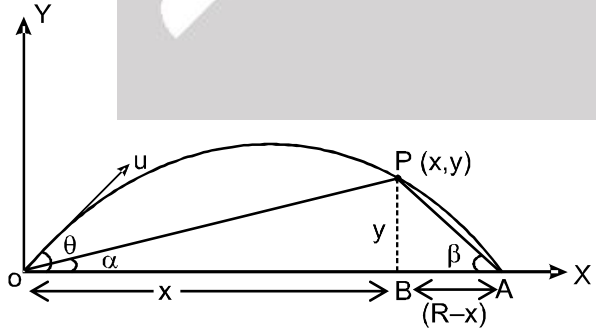
x (b + a) = $\left(\frac{b\sin 2\alpha + a\sin 2\beta}{g}\right)u^2$

Substituting the value of x from eq. (3), we get

$\frac{u^2 \sin 2\theta}{g}$ (b + a) = $\left(\frac{b\sin 2\alpha + a\sin 2\beta}{g}\right)u^2$

Solving this equation, we will get θ.

2. The situation is shown in the fig.



From fig tan α + tan β = $\frac{y}{x} + \frac{y}{(R-x)}$

where R is the range.

$\therefore$ tan α + tan β = $\frac{y(R-x) + xy}{x(R-x)}$

or $\tan\alpha + \tan\beta = \frac{y}{x} \times \frac{R}{(R-x)}$ ....(1)

but $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

or $\tan\theta = \frac{y}{x} \times \frac{R}{(R-x)}$ ....(2)

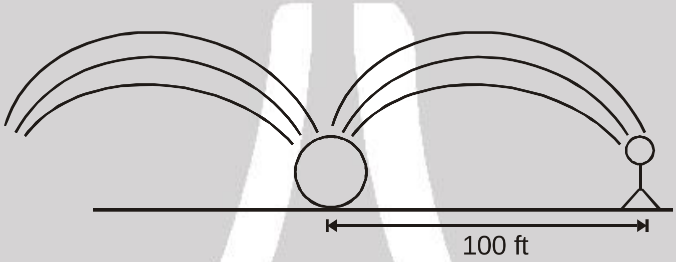
From equations (1) and (2), we have

$\tan\theta = \tan\alpha + \tan\beta$.

3. According to given problem u = 80 f / s

Range = $\frac{u^2 \sin 2\theta}{g}$

$\sin 2\theta = \frac{100 \times 32}{(80)^2} = 1/2$



$\theta = 15°$ For same Range $\theta = 15°, 75°$

Thus there will be two time of flight

$T_1 = \frac{2u\sin 15°}{g} = \frac{2 \times 80 \times \sin 15°}{32}$ (minimum time)

$\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

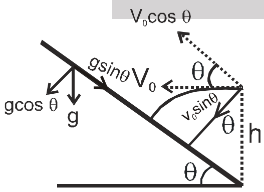
$T_2 = \frac{2u\sin 75°}{g} = \frac{2 \times 8 \times \sin 75°}{32}$ (maximum time)

$\sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

Danger time = Maximum time – Minimum time = $(T_2 - T_1)$

$= \frac{2 \times 80}{32}$ [sin75° –sin 15°] $= \frac{2 \times 80}{32}\left[\frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}\right] = \frac{5}{\sqrt{2}}$ sec.

4.



Parallel to plane

$0 = V_0\cos\theta - g\sin\theta \times t$

$t = \frac{V_0\cos\theta}{g\sin\theta}$ ...... (1)

Perpendicular to plane

$h\cos\theta = V_0 \sin\theta\, t + \frac{1}{2}(g\cos\theta)\, t^2$

$h\cos\theta = V_0 \sin\theta\left(\frac{V_0\cos\theta}{g\sin\theta}\right) + (\frac{1}{2} g\cos\theta)\left(\frac{V_0\cos\theta}{g\sin\theta}\right)^2$

$h\cos\theta = V_0^2\,\frac{\cos\theta}{g} + \frac{V_0^2\cos\theta\cot^2\theta}{2g}$

$h = \frac{V_0^2}{g} + \frac{V_0^2\cot^2\theta}{2g}$

$2gh = (2 + \cot^2\theta)\, V_0^2$

$V_0 = \sqrt{\frac{2gh}{2+\cot^2\theta}}$

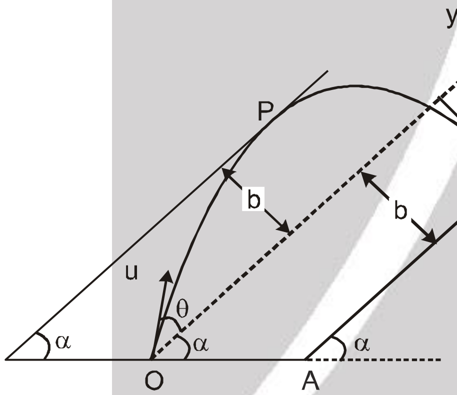
5. Consider the motion of the particle from O to P.

The velocity v_y at P is zero.

$v_y^2 = u_y^2 + 2a_y s_y$

$\therefore \quad 0 = (u\sin\theta)^2 - 2(g\cos\alpha)\, b$

or $\quad b = \frac{u^2\sin^2\theta}{2g\cos\alpha}$ ....(i)



Now, consider the motion of the particle from O to Q.

The particle strikes the point Q at 90º to AB, i.e., its velocity along x-direction is zero.

Using $\quad v_x = u_x + a_x t$, we have

$0 = u\cos\theta - (g\sin\alpha)t$

or $\quad t = \frac{u\cos\theta}{g\sin\alpha}$ ...(ii)

For motion in y-direction, $s_y = u_y t + \frac{1}{2} a_y t^2$

or $\quad -b = u\sin\theta\left(\frac{u\cos\theta}{g\sin\alpha}\right) + \frac{1}{2}(-g\cos\alpha)\left(\frac{u\cos\theta}{g\sin\alpha}\right)^2$ ...(iii)

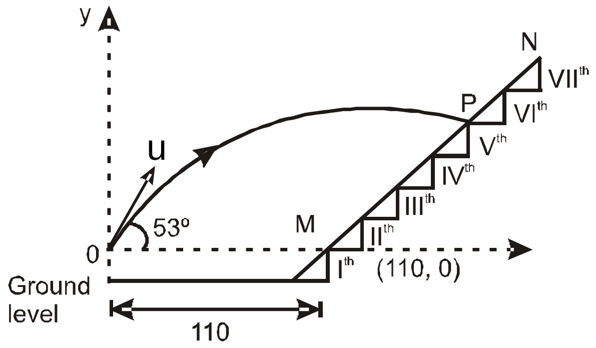
From Eqs. (i) and (iii)

or $\quad -\frac{u^2\sin^2\theta}{2g\cos\alpha} = \frac{u^2\sin\theta\cos\theta}{g\sin\alpha} - \frac{gu^2\cos\alpha\cos^2\theta}{2g^2\sin^2\alpha}$

or $\quad -\frac{\sin^2\theta}{2\cos\alpha} = \frac{\sin\theta\cos\theta}{\sin\alpha} - \frac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$

Solving, we get $\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$

6. Equation of ball,



$$y = x \tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Substituting the values,

$y = 1.33\,x - 0.011\,3x^2$ .....(1)

Slope of line MN is 1 and it passes through point (110 m, 0). Hence the equation of this line can be written as,

$y = x - 110$ ....(2)

Point of intersection of two curves is say P. Solving (1) and (2) we get positive value of y equal to 4.5 m.

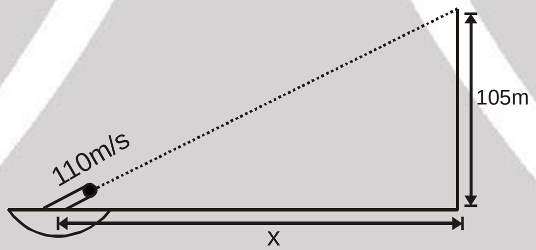
i.e., $y_p = 4.5$

Height of one step 1 m. Hence, the ball will collide somewhere between

y = 4 m and y = 5 m. Which comes out to be 6th step.

7. $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

$105 = x\tan\theta - \frac{5}{(110)^2}x^2(1+\tan^2\theta)$



$\frac{5x^2}{(110)^2}\tan^2\theta - x\tan\theta + \left[105 + \frac{5x^2}{(110)^2}\right] = 0$ $(b^2 - 4ac > 0)$

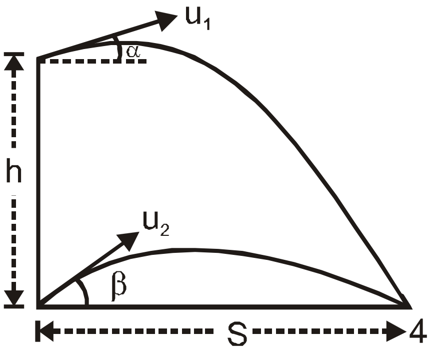
$x^2 - 4 \times \frac{5x^2}{110^2}\left(105 + \frac{5x^2}{110^2}\right) > 0$

$1 - 20 \times \frac{105}{110^2} - \frac{100x^2}{(110)^4} > 0$

On solving we get

x = 1100 m.

8.



$-h = (u_1 \sin\alpha)\, t - \frac{1}{2} g t^2$

$0 = (u_2 \sin\beta)\, t - \frac{1}{2} g t^2$

$\therefore \quad (u_1 \sin\alpha)t + h = (u_2 \sin\beta)t$

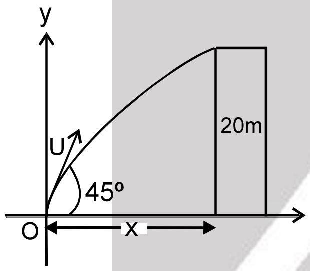
But $\quad t = \frac{s}{u_1 \cos\alpha} = \frac{s}{u_2 \cos\beta}$

$(u_1 \sin\alpha)\left(\frac{s}{u_1 \cos\alpha}\right) + h = (u_2 \sin\beta)\left(\frac{s}{u_2 \cos\beta}\right)$

$h + s \tan\alpha = s \tan\beta$

$h = s(\tan\beta - \tan\alpha)$.

9.



Let us assume that person throws ball from distance x. Assuming point of projection as origin

$y = x \tan\theta - \frac{1}{2} \frac{g x^2}{u^2 \cos^2\theta}$

$20 = x \tan 45^{\circ} - \frac{1}{2} \frac{10 x^2}{u^2 \cos^2 45}$

$u^2 = \frac{10x^2}{x - 20}$ ....(1)

For 'u' to be minimum du/dx = 0

On differentiating w.r.t. 'x'

$2u \frac{du}{dx} = \frac{10 \times 2x \ (x - 20) - 10x^2 \times 1}{(x-20)^2} = 0$

du/dx = 0

$\Rightarrow \quad 20x(x - 20) - 10x^2 = 0$

$20x^2 - 10x^2 - 400x = 0$

$10x(x - 40) = 0$

$x = 40$

For x greater than 40, slope is positive & x less than 40, slope is negative

So at x = 40 There is a minima

Required minimum velocity from equation (1)

$u^2_{min} = \dfrac{10 \times 40^2}{40 - 20}$

$u_{min} = \sqrt{800}$

u_{min} = $20\sqrt{2}$ m/s

10. Along Horizontal direction

$x = v_0 \cos 53^o\, t = v_0 \cos 37^o\, (t - t_0)$

$\dfrac{3}{5}\, t = \dfrac{4}{5}\,(t - t_0) \quad \Rightarrow \quad 3t = 4\,(t - t_0)$ ....(1)

Vertical direction ;

$y = v_0 \sin 53^o\, t - \dfrac{1}{2} g t^2 = v_0 \sin 37^o\,(t - t_0) - \dfrac{1}{2} g\,(t - t_0)^2$ ....(2)

$v_0 \times \dfrac{4}{5}\, t - 5t^2 = v_0 \times \dfrac{3}{5} \times \dfrac{3t}{4} - \dfrac{10}{2} \times \dfrac{9t^2}{16}$

$\dfrac{v_0 t}{5}\left(4 - \dfrac{9}{4}\right) = 5\, t^2\left(1 - \dfrac{9}{16}\right)$

$\dfrac{250}{5} \times \dfrac{7}{4} = 5t \times \dfrac{7}{16} \quad \Rightarrow \quad t = 40 \quad$ so $\quad t_0 = 10$ sec

11. Let the speed of shell be u and the speed of wind be v.

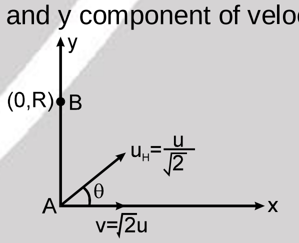
The time of flight T remains unchanged due to windstorm

$T = \dfrac{\sqrt{2}\,u}{g}$ ....(1)

Horizontal component of velocity of shell in absence of air

$u_H = \dfrac{u}{\sqrt{2}}$ ....(2)

Hence the net x and y component of velocity of shell (see figure) are



$u_x = \sqrt{2}\,u + \dfrac{u}{\sqrt{2}} \cos\theta$ ....(3a)

$u_y = \dfrac{u}{\sqrt{2}} \sin\theta$ ....(3b)

∴ The x and y coordinate of point P where shell lands is

$x = u_x T = \left(\sqrt{2}\,u + \dfrac{u}{\sqrt{2}} \cos\theta\right) \dfrac{\sqrt{2}u}{g} = 2R + R\cos\theta$ ....(4a)

$y = u_y T = \left(\dfrac{u}{\sqrt{2}} \sin\theta\right) \dfrac{\sqrt{2}u}{g} = R\sin\theta$ ....(4b)

∴ The distance S between B and P is given by

$S^2 = (x - 0)^2 + (y - R)^2 = (2R + R\cos\theta)^2 + (R\sin\theta - R)^2$

$= R^2 [6 + 4\cos\theta - 2\sin\theta]$

$= R^2 \left[6 + \sqrt{20}\left(\frac{4\cos\theta}{\sqrt{20}} - \frac{2\sin\theta}{\sqrt{20}}\right)\right]$

$\therefore \quad S_{minimum} = R\sqrt{6-\sqrt{20}}$

$= R\sqrt{6-2\sqrt{5}}$ or $R(\sqrt{5}-1)$ **Ans.**

Alternate : Circle in fig. (1) represents locus of all points where shell lands on the ground in absence of windstorm.

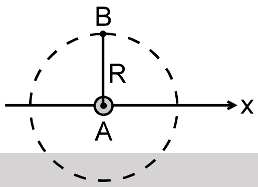


fig.1

Let the speed of shell be 'u' and the speed of wind be $v = \sqrt{2}\,u$. Let T be the time of flight, which remains unaltered even when the windstorm blows. Since R is the maximum range angle of projection is 45º with the horizontal.

Then $R = \frac{u}{\sqrt{2}} T$ ....(1)

As a result of flow of wind along x-axis, there is an additional shift (Δx) of the shell along x-axis in time of flight.

$\Delta x = vT = \sqrt{2}\,uT = 2R.$

Hence locus of all points where shell lands on ground shifts along x-axis by 2R as shown in fig. (2).

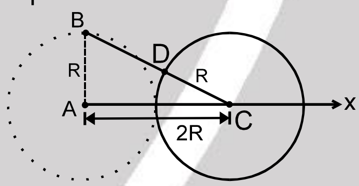


fig.2

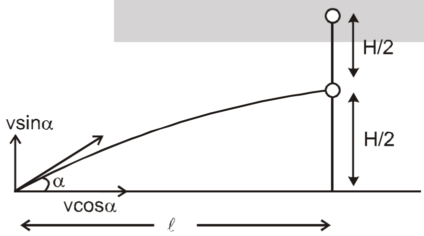
From the fig (2).

$BC = \sqrt{R^2 + (2R)^2} = \sqrt{5R^2} = \sqrt{5}\,R$

Hence the minimum required distance is

$BD = BC - DC = \sqrt{5}\,R - R =$ **$(\sqrt{5} - 1)$ R Ans.**

12.



Let time taken in collison is t. Then $\frac{H}{2} = \frac{1}{2}gt^2$ ...(i)

also for projetile motion

$\ell = (v\cos\alpha)t$ ...(ii)

and $\frac{H}{2} = (v\sin\alpha)t - \frac{1}{2}gt^2$ ...(iii) from (i) and (iii)

$\frac{H}{2} = vt\sin\alpha - \frac{H}{2}$

$\Rightarrow$ H = vt sinα ...(iv)

from (ii) & (iv)

$\frac{H}{\ell} = \tan\alpha$

$v = \frac{H}{t\sin\alpha} = \frac{H}{\sqrt{\frac{H}{g}}}$ $(\because t = \sqrt{\frac{H}{g}}$ from (i))

and $\sin\alpha = \frac{H}{\sqrt{\ell^2 + H^2}}$

So $V = \sqrt{\frac{\ell^2 + H^2}{H/g}} = \sqrt{\frac{g}{H}(\ell^2 + H^2)} = \sqrt{\frac{gH^2}{H}\left(\frac{\ell^2}{H^2}+1\right)} = \sqrt{gH\left(1+\frac{\ell^2}{H^2}\right)}$

13. $u = 5\sqrt{3}$ m/s.

$\therefore$ $u\cos 60° = \frac{5\sqrt{3}}{2}$ m/s

and तथा $u \sin 60^0 = 5\sqrt{3} \times \frac{\sqrt{3}}{2}$ = 7.5 m/s

Since the horizontal displacement of both the shots are equal , the second should be fired early because its horizontal component of velocity u cos60^0 is less than the other's which is u or $5\sqrt{3}$ m/s.

Now let first shot takes t_1 time to reach the point P and the second t_2. Then –

$x = (u\cos 60^o)\, t_2 = u.\, t_1$

or $x = \left(\frac{5\sqrt{3}}{2}\right)t_2 = 5\sqrt{3}\, t_1$ ....(1)

or $t_2 = 2t_1$ ....(2)

and $h = \frac{1}{2}g\, t_1^2 = \frac{1}{2}gt_2^2 - (7.5)\, t_2$

Taking g = 10 m/s^2 लेने पर

$h = 5t_2^2 - 7.5\, t_2 = 5t_1^2$ ....(3)

Substituting $t_2 = 2t_1$ in equation (3), we get

$5(2t_1)^2 - 7.5\,(2\,t_1) = 5t_1^2$

or $5t_1^2 = 5t_1$

$t_1 = 0$ and 1s

Hence t_1 = 1s and

$t_2 = 2t_1 = 2s$

$x = 5\sqrt{3}\, t_1 = 5\sqrt{3}$ m (From equation I)

and $h = 5\, t_1^2 = 5\,(1)^2 = 5$ m (From equation 3)

$\therefore$ y = 10 – h = (10 – 5) = 5 m

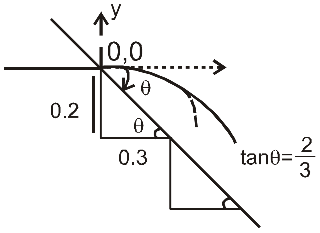
Hence

(i) Time interval between the firings = $t_2 - t_1$ = (2 – 1) s

Δ = 1s

(ii) Coordinates of point P = (x, y) = $5\sqrt{3}$ m. 5 m

14. We have the point of projection as (0, 0)
We have the equation of straight line (as shown in fig.)



$y = x \tan\theta$

$y = \frac{-2}{3} \times$ ....(1)

Also the equation of trajectory for horizantal projection

$y = \frac{-1}{2} g \frac{x^2}{u^2}$ ....(2)

from (1) and (2)

$\frac{1}{2} g \frac{x^2}{u^2} = \frac{2}{3} x$

or $x = \frac{2}{3} \times \frac{u^2}{5} = \frac{2}{3} \times \frac{4.5 \times 4.5}{5} = 3 \times 0.9$

If no. of steps be n then $n \times 0.3 = 3 \times 0.9$

n = 9

15. $x = y^2 + 2y + 2$

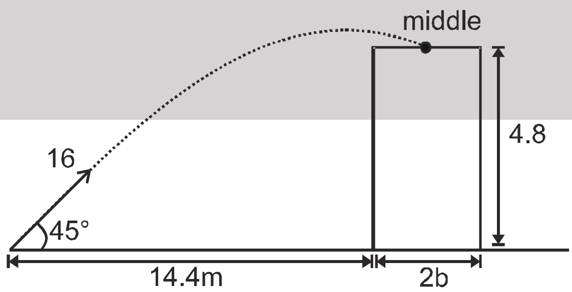
$\frac{dx}{dt} = 2y\frac{dy}{dt} + 2\frac{dy}{dt} + 0$

$\frac{d^2x}{dt^2} = 2\left(\frac{dy}{dt}\right)^2 + 2y\frac{d^2y}{dt^2} + 2\frac{d^2y}{dt^2}$

$\frac{d^2y}{dt^2} = 0.$ ($\frac{dy}{dt}$ =5 m/s)

$\frac{d^2x}{dt^2} = 2(5^2) + 0 + 0 = 50$ m/s^2. **Ans. "A"**

16. Equation of Trajectory



$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

4.8 = (14.4 + b) × 1 – 1/2

on solving this equation

we get 2b = 9.6 m ⇒ b = 4.8 m

Now to find angle of projection for projectile having speed $10\sqrt{3}$ m/s.

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$ [$x = 2b + 14.4 \Rightarrow 9.6 + 14.4 = 24$ m]

$4.8 = 24\tan\theta - \frac{1}{2}\frac{10(24)^2\sec^2\theta}{(10\sqrt{3})^2}$

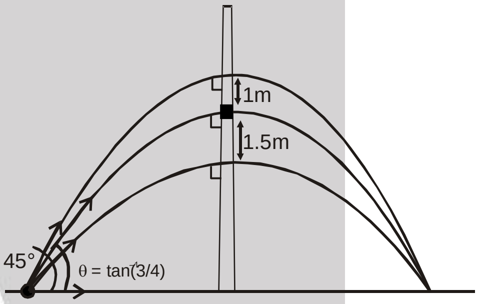
$4.8 = 24\tan\theta - \frac{24\times 4}{10}(1+\tan^2\theta)$

$4\tan^2\theta - 10\tan\theta + 6 = 0$

$\tan\theta = \frac{3}{2}, 1$

$\theta = \tan^{-1}\frac{3}{2}, \theta = 45°$

17. $H = \frac{u^2\sin^2\theta}{2g}$, $R = \frac{u^2\sin 2\theta}{g}$



so, $\frac{H}{R} = \left(\frac{\tan\theta}{4}\right)$ ......(1)

$\frac{H+1}{R} = \left(\frac{\tan 45°}{4}\right) = \frac{1}{4}$ ....(2)

$\frac{H-1.5}{R} = \frac{\tan\left[\tan^{-1}(3/4)\right]}{4}$

$\frac{H-1.5}{R} = \frac{3/4}{4} = \frac{3}{16}$ ....(3)

$\frac{H+1}{H-1.5} = \frac{4}{3} \Rightarrow \frac{10}{R} = \frac{1}{4}$ $\Rightarrow$ **R = 40 m**

$3H + 3 = 4H - 6$

$H = 9$ m

$\frac{9}{40} = \frac{\tan\theta}{4}$

$\tan\theta = \frac{9}{10}$ $\Rightarrow \theta = \tan^{-1}\left(\frac{9}{10}\right)$

$R = 40$ $\tan\theta = 9/10$

$\frac{u^2\sin 2\theta}{g} = 40$ $\sin\theta = \frac{9}{\sqrt{181}}$

Using $R = \frac{u^2.2\sin\theta\cos\theta}{g}$

$\frac{u^2 2\left(\frac{9}{\sqrt{181}}\right)\left(\frac{10}{\sqrt{181}}\right)}{10} = 40$

$u^2 = \frac{3620}{9} \Rightarrow u = \frac{\sqrt{3620}}{3}$ m/s